Annals of Natural Sciences
(Peer Reviewed and UGC Approved International Journal)
Vol. 3(4), December 2017: 94-103
Journal's URL: http://www.crsdindia.com/ans.html
Email: crsdindia@gmail.com

e-ISSN: 2455-667X

Annals of Natural Sciences

ORIGINAL ARTICLE

MHD Convective Flow of Viscous Fluid through a Porous Medium Bounded by an Oscillating Porous Plate in SLIP Flow Regime with Mass Transfer

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ABSTRACT

An analysis of velocity, temperature and mass distribution, skin friction and rate of heat and Ma transfer of the flow of a viscous incompressible fluid of small electrical conductivity in a porous medium near an oscillating infinite porous flat plate in slip flow regime under influence of a transverse magnetic field of uniform strength. The effects of Modified Grasholf number (G'), Hartmann number (M) and Rarefaction parameter (R) on velocity and skin friction are discussed with the help of graphs and tables. The effect of Schmidt number (5) on concentration of fluid is also discussed.

Key words: *MHD Convective Flow, Viscous Fluid, Oscillating Porous Plate, Mass Transfer*

Received: 1st Sept. 2017, Revised: 7th Nov. 2017, Accepted: 20th Nov. 2017

How to cite this article:
Dwivedi R.K. and Varshney N.K. (2017): MHD Convective Flow of Viscous Fluid through a Porous Medium Bounded by an Oscillating Porous Plate in SLIP Flow Regime with Mass Transfer. Annals of Natural Sciences, Vol. 3[4]: December, 2017: 94-103.

INTRODUCTION

Many papers have been published on the theory of laminar boundary layers in unsteady flow. The study of fluctuating flow is important in the paper industry and many other technological fields. Due to this reason many research workers have paid theirattention towards the fluctuating flow of viscous, incompressible fluid past an infinite plate. Stokes (1901) and Rayleigh (1911) studied the flow of a viscous and incompressible fluid about an infinite flat wall which executelinear harmonic oscillation parallel to itself. Sturart (1955) investigated the response of skin friction and temperature of anfig infinite plate thermometer, to fluctuations in the stream with suction tit the plate, Ong and Nicholls (1959) extended the method to obtain the flow in magnetic field near an infinite flat wall which oscillates in its own plane. Ahmadi and Manvi (1971) have derived a general equation of motion for viscous flow through a rigid porous medium and applied to some basic flow problems. Yamamoto and Iwamura investigated the flow with convective acceleration through a porous medium. Gupta and Babu (1987) studied the flow of a viscous incompressible fluid through a porous medium near an oscillating infinite porous flat plate in the slip flow regime. Recently Singh (2000) studied MHD convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime.

This work is an extension of work Singh (2000) with mass transfer. The aim of present investigation is to study the effect of mass transfer on MHD convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime.

FORMULATION OF THE PROBLEM

Let us denote velocity components u, v in direction of x, y respectively and temperature by T and concentration by C. Under these, assumptions, the physical variables are functions of y and t only. A uniform magnetic field B. is acting along the y-axis forproblemaeronautical engineering the Reynold number is usually' small, mutt's* such conditions the induced magnetic field due to the flow may he neglected with respect to the applied field, the pressure fluid is assumed constant, if V_0represents the velocity of suction or injection at the plate, the equation of continuity is:

$$\frac{\partial v}{\partial y} = 0 \quad \ldots\ldots \quad \ldots\ldots \quad (1)$$

with the condition y = 0, v leads to the result $v = V_o (< 0)$.

The boundary layer equations are:

$$\frac{\partial u}{\partial t} + V_o \frac{\partial u}{\partial y} = g\beta (T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \sigma \frac{B_o^2}{\rho} u + g\beta'(C - C_\infty) - \frac{\nu}{K} u \quad \ldots\ldots \quad \ldots\ldots \quad (2)$$

$$\frac{\partial T}{\partial t} + V_o \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad \ldots\ldots \quad \ldots\ldots \quad (3)$$

$$\frac{\partial C}{\partial t} + V_o \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad \ldots\ldots \quad \ldots\ldots \quad (4)$$

Where p is the density, g is the acceleration due to cavity, 13 is the coefficient of volume expansion, β' is the coefficient of concentration expansion, v is the Kinematic viscosity, T_α and C. are the temperature and concentration of the fluid in the free stream, CY_α is the electric conductivity, B_o is the magnetic induction, K is porosity parameter, of is the thermal diffusivity, D is the concentration diffusivity.

First order velocity slip boundary conditions of the problem when the plate execute linear harmonic oscillation in its own plane is given by Street (1960) are-

$$u = U_o \cos(nt) + L_1 \partial u/\partial y, \quad T - T_\infty = (T_w - T_\infty)\cos(nt) + L_1 \partial T/\partial y$$
$$C - C_\infty = (C_w - C_\infty)\cos(nt) + L_1 \partial C/\partial y \quad \text{at} \quad y = 0 \quad \ldots\ldots (5)$$
$$u \to 0 \quad T \to 0 \quad C \to 0 \quad \text{as} \quad y \to \infty$$

where $L_1 = (2-m_1)L/m_1$and $L = \mu[\pi/(2p\rho)]^{1/2}$ is the mean free path and m_1 is the Maxwell's reflection coefficient.

On introducing the following non dimensional quantities

$$y^* = U_o \frac{y}{\nu} \qquad u^* = \frac{u}{U_o} \qquad \theta^* = \frac{T - T_\infty}{T_w - T_\infty} \qquad \phi^* = \frac{C - C_\infty}{C_w - C_\infty}$$

$$t^* = U_o^2 \frac{t}{\nu} \qquad V_o^* = \frac{V_o}{U_o} \qquad n^* = \frac{\nu n}{U_o^2} \qquad K^* = \frac{K U_o^2}{\nu}$$

$$R = U_o \frac{L_1}{\nu} \text{ (Rarefraction parameter)}, \quad S_c = \frac{\nu}{D} \text{ (Schimdt No.)}$$

$$M = \frac{B_o}{U_o}\left(\frac{\nu\sigma}{\rho}\right)^{1/2} \text{ (Hartmann number)}, \quad P_r = \frac{\nu}{\alpha} \text{ (Prandtl No.)}$$

$$G = \frac{g\beta\nu(T_w - T_\infty)}{U_o^3} \text{(Grasholf No)}, \; G' = \frac{g\beta'\nu(C_w - C_\infty)}{U_o^3} \text{(Mod.Gr. No)}$$

Equations (2), (3) and (4) after dropping the asterisks (*) can be written as

$$\frac{\partial u}{\partial t} + V_o \frac{\partial u}{\partial y} = G\theta + G'\phi + \frac{\partial^2 u}{\partial y^2} - \frac{u}{K} - M^2 u \qquad \dots \qquad (6)$$

$$P_r\left(\frac{\partial \theta}{\partial t} + V_o \frac{\partial \theta}{\partial y}\right) = \frac{\partial^2 \theta}{\partial y^2} \qquad \dots \qquad \dots \qquad (7)$$

$$S_c\left(\frac{\partial \phi}{\partial t} + V_o \frac{\partial \phi}{\partial y}\right) = \frac{\partial^2 \phi}{\partial y^2} \qquad \dots \qquad \dots \qquad (8)$$

with the boundary conditions :

$$\begin{array}{ll} u = \cos(nt) + R\,\partial u/\partial y, \quad \theta = \cos(nt) + R\,\partial\theta/\partial y & \\ \phi = \cos(nt) + R\,\partial\phi/\partial y & \text{at} \quad y = 0 \\ u \to 0, \quad \theta \to 0, \quad \phi \to 0 & \text{as} \quad y \to \infty \end{array} \qquad \dots \qquad (9)$$

The solutions of equations (6), (7) and (8) are :

$$u = F(y)\cos(nt - Ay) \qquad \dots \qquad \dots \qquad (10)$$

$$\theta = g(y)\cos(nt - Ay) \qquad \dots \qquad \dots \qquad (11)$$

$$\phi = h(y)\cos(nt - Ay) \qquad \dots \qquad \dots \qquad (12)$$

where A is a constant. Its value will be determined later on.

Putting for u in (6), for θ in (7) and for φ in (8), and equating the coefficient of sin and cos terms on the both sides, we get

for u: $2AF' - (AV_o - n)F = 0$ .... (13)

$F'' - V_o F' - (A^2 + 1/K + M^2)F + G\theta + G'\phi = 0$ .... (14)

for θ: $2Ag' - P_r(AV_o - n)g = 0$ .... (15)

$g'' - P_r V_o g' - A^2 g = 0$ .... (16)

for φ: $2Ah' - S_c(AV_o - n)h = 0$ .... (17)

$h'' - S_c V_o h' - A^2 h = 0$ .... (18)

with the boundary conditions

$$[1-AR\tan(nt)]\,F(y) = 1 + RF'(y) \text{ at } y = 0$$
$$F(y) \to 0 \text{ as } y \to \infty \quad \ldots \quad \ldots \quad (19)$$
$$[1-AR\tan(nt)]\,g(y) = 1 + Rg'(y) \text{ at } y = 0$$
$$g(y) \to 0 \text{ as } y \to \infty \quad \ldots \quad \ldots \quad (20)$$
$$[1-AR\tan(nt)]\,h(y) = 1 + Rh'(y) \text{ at } y = 0$$
$$h(y) \to 0 \text{ as } y \to \infty \quad \ldots \quad \ldots \quad (21)$$

where dashes denote the differentiation with respect to y.

The solution of the equation (13) applying the boundary conditions (19) is

$$F(y) = \frac{\exp[-0.5(nA^{-1} - V_o)y]}{1 + 0.5R(nA^{-1} - V_o) - AR\tan(nt)} \quad \ldots \quad (22)$$

Putting for F in equation (14),

$$4A^4 + SA^2 - n^2 = 0$$

where $S = [\,V_o^2 + 4(1/K + M^2 - G - G')]$

Since A^2 is to remain positive, hence

$$A = \left(\frac{-S + (S^2 + 16n^2)^{1/2}}{8} \right)^{1/2} \quad .... \quad .... \quad (23)$$

To obtain u eliminate F(y) from (10) & (22), we get

$$u = \frac{\exp[-0.5(nA^{-1} - V_o)y] \cos(nt - Ay)}{1 + 0.5R(nA^{-1} - V_o) - AR\tan(nt)} \quad .... \quad (24)$$

The solution of the equation (15) applying the boundary conditions (20) is

$$g(y) = \frac{\exp[-0.5 P_r(nA^{-1} - V_o) y]}{1 + 0.5RP_r(nA^{-1} - V_o) - AR\tan(nt)} \quad .... \quad (25)$$

Putting for g in equation (16),

$$4A^4 + S_1A^2 - n^2 P_r^2 = 0$$

where $\quad S_1 = V_o^2 P_r^2$

Since A^2 is to remain positive, hence

$$A = \left(\frac{-S_1 + (S_1^2 + 16n^2 P_r^2)^{1/2}}{8} \right)^{1/2} \quad .... \quad .... \quad (26)$$

To obtain θ eliminate g(y) from (11) & (25), we get

$$\theta = \frac{\exp[-0.5P_r(nA^{-1} - V_o)y] \cos(nt - Ay)}{1 + 0.5RP_r(nA^{-1} - V_o) - AR\tan(nt)} \quad .... \quad (27)$$

The solution of the equation (17) applying the boundary conditions (21) is

$$h(y) = \frac{\exp[-0.5 S_c(nA^{-1} - V_o) y]}{1 + 0.5RS_c(nA^{-1} - V_o) - AR\tan(nt)} \quad .... \quad (28)$$

Putting for h in equation (18),

$$4A^4 + S_2A^2 - n^2 S_c^2 = 0$$

where $S_2 = V_o^2 S_c^2$

Since A^2 is to remain positive, hence

$$A = \left(\frac{-S_2 + (S_2^2 + 16n^2 S_c^2)^{1/2}}{8} \right)^{1/2} \quad .... \quad .... \quad (29)$$

To obtain ϕ eliminate h(y) from (12) & (28), we get

$$\phi = \frac{\exp[-0.5S_c(nA^{-1} - V_o)y] \cos (nt - Ay)}{1 + 0.5RS_c (nA^{-1}-V_o)-AR \tan(nt)} \quad .... \quad (30)$$

We can calculate the expression for the skin friction, the rate of heat transfer and rate of mass transfer as

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=o} = \frac{A \sin (nt) - 0.5(nA^{-1} - V_o) \cos (nt)}{1 + 0.5R(nA^{-1}-V_o)-AR \tan(nt)} \quad .... \quad (31)$$

$$q_1 = -\left(\frac{\partial \theta}{\partial y}\right)_{y=o} = \frac{A \sin (nt) - 0.5P_r(nA^{-1} - V_o) \cos (nt)}{1 + 0.5RP_r(nA^{-1}-V_o)-AR \tan(nt)} \quad .... \quad (32)$$

$$q_2 = -\left(\frac{\partial \phi}{\partial y}\right)_{y=o} = \frac{A \sin (nt) - 0.5S_c(nA^{-1} - V_o) \cos (nt)}{1 + 0.5RS_c(nA^{-1}-V_o)-AR \tan(nt)} \quad .... \quad (33)$$

RESULTS AND DISCUSSION

Fluid Velocity Profile of boundary layer flow is tabulated in Table- I and plotted in Fig.-I having five Graphs for n = 2, V_0 =-2, t = 0, G' = 5, K = 1 and following different values of R,M and G.

	R	M	G'
For Graph- I	0.2	1	0
For Graph- II	0.2	1	2
For Graph- III	0.2	1	4
For Graph- IV	0.6	1	2
For Graph- V	0.2	3	2

It is observed from Fig. I that all velocity Graphs are decreasing sharply and have minimum value (below y-axis) at y = 1.2 then after velocity in each Graphs begins to increase and tends to zero with the increasing y.

It is also observed from Fig- I that velocity decreases with the increase in G' but it decreases (upto y=0.5) with the increase in M, after it velocity increases. Velocity also decreases upto y = 0.9 with the increases in R, after it velocity increases. These effects of above parameters are noticed upto y = 3 after it these effects are negligible.

Concentration profile is tabulate in Table- II and plotted in Fig- II having Graphs from I to III for n = 2, V_0 = -2 t = 0 and following different values of S_c.

	S_c
For Graph- I	0.2
For Graph- II	0.4
For Graph- II	0.6

It is inferred from Fig.- II that concentration in all graph decreases sharply till y =3, after it concentration reduces to zero.

It is also inferred that concentration fluid decreases with the increase in S_c.

Skin friction Profile is tabulated in Table- III and plotted in Fig. III having five Graphs. It is observed that all skin friction Graphs are sinusoidal in nature.

It is also observed that skin friction increases numerically with the increases in G' and M, but it decrease numerically with increase in R.

Table-I : Values of velocity u for Fig.-I when V_0 = -2, n =2, G = 5, t = 0, K = 1 and different values of R, M and G'

y	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V
0	0.7524	0.7710	0.7809	0.5287	0.6218
1	0.0030	-0.0820	-0.1514	-0.0563	0.0262
2	-0.0280	-0.0221	0.0116	-0.0151	0.0008
3	-0.0003	0.0089	0.0047	0.0061	0.0000
4	0.0010	-0.0008	-0.0025	-0.0005	0.0000

Table-II : Values of concentration ϕ for Fig.-II when V_0 = -2, n =2, t = 0 and different values of S_c

y	Graph-I	Graph-II	Graph-III
0	1	1	1
1	0.4674	0.2798	0.1757
2	0.1725	0.0461	0.0115
3	0.0377	-0.0051	-0.0048
4	-0.0104	-0.0079	-0.0023
5	-0.0197	-0.0039	-0.0006

Table-III: Values of skin friction τ for Fig.-III when V_0 = -2, n =2, G = 5, K = 1 and different values of R, M and G′

t	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V
0	-1.2379	-1.1452	-1.0957	-0.7854	-1.8909
1	0.7355	0.9145	1.0700	0.4376	0.8940
2	0.4298	-0.2280	-0.9943	-0.4063	1.1272
3	-1.2843	-1.3083	-1.3651	-0.8236	-1.8522
4	0.3201	0.4221	0.4989	0.1680	0.3685

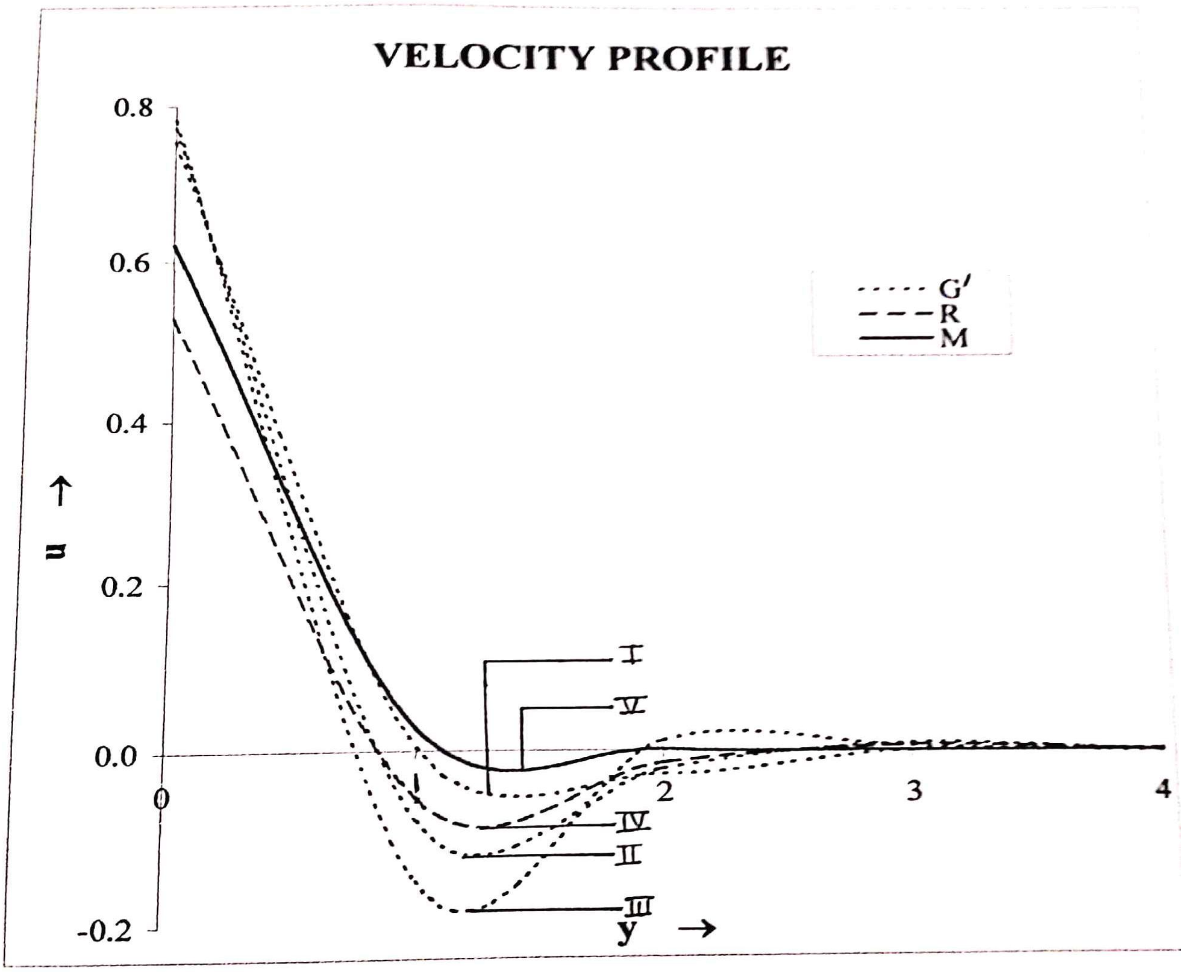

Fig. 1: Valocity Profile

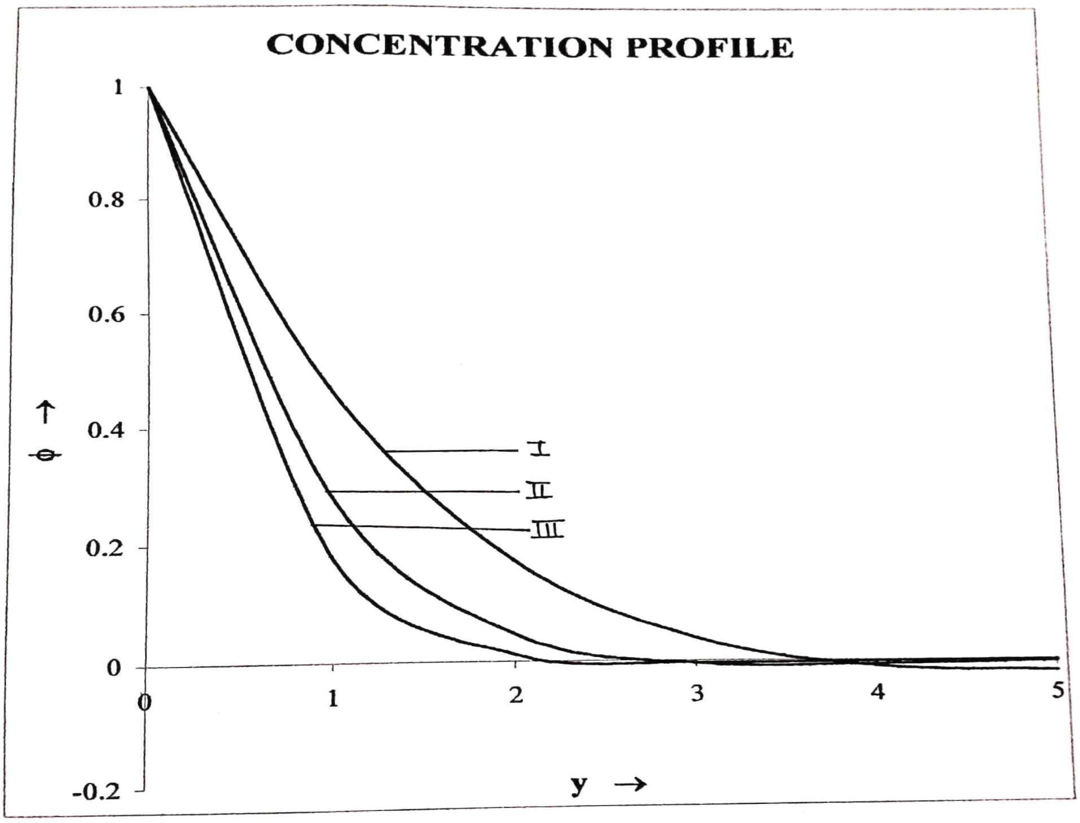

Fig. 2: Concentration Profile

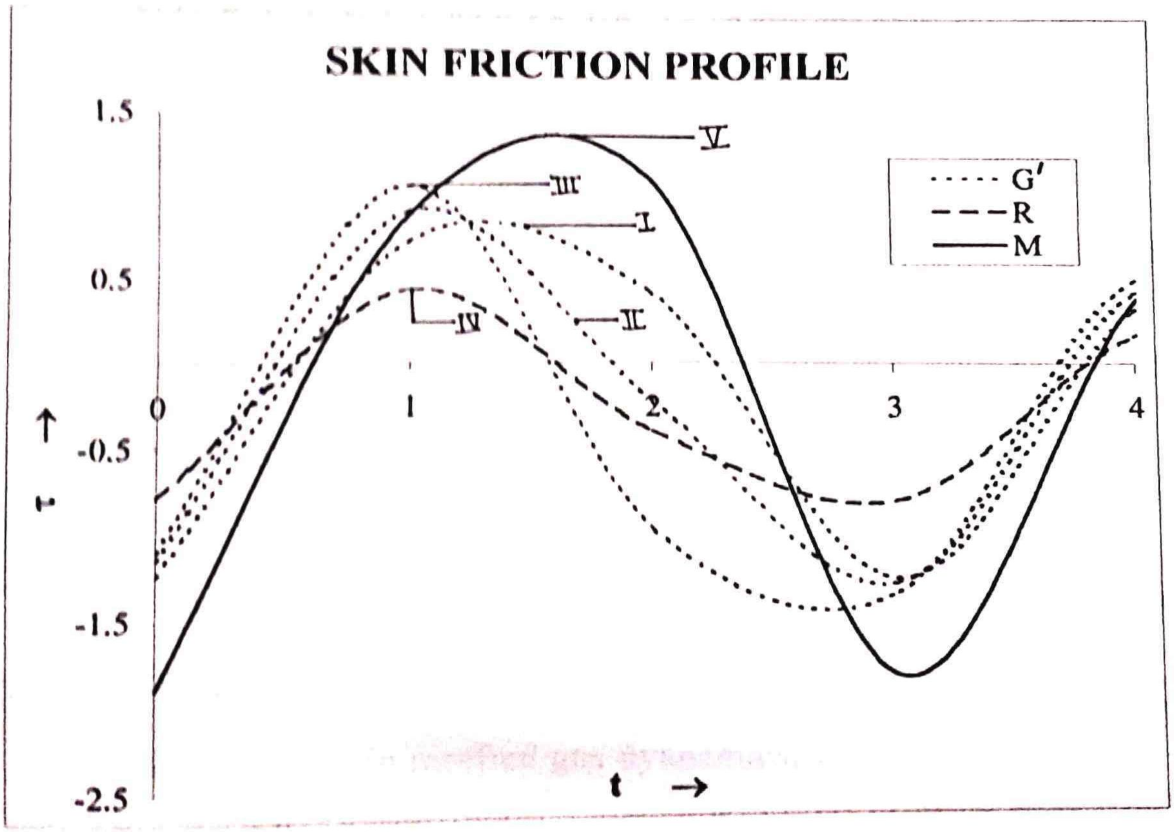

Fig. 3: Skin Friction Profile

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