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ORIGINAL ARTICLE

The M*/G/1 Queueing System with and without Vacation Time under Non-Preemptive Last-Come-First-Served (LCFS)

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ABSTRACT

In the present chapter we have considered M/G/1 queue with and without vacation time under non-preemptive last-come-first-served (LCFS). The Laplace Stieltjes transform of the distribution function of the steady state waiting time is found. The first two moments of the waiting time are obtained. We find the relationship between second moments of the waiting time in the M*/G/1 queue with and without vacation time under first-come-first-serve (FCFS) and LCFS.*

Key words: *The M*/G/1, Queueing System, Vacation Time, Multiple vacation, Single Vacation, Size*

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INTRODUCTION

In the present chapter we study the batch arrival M*/G/1 queue with and without vacation time under non-preemptive last-come-first-served (LCFS). In some queuing systems, the server takes a vacation of random length each time the system becomes empty. The vacations time is utilized for some additional word. If the server returns from a vacation and he finds one or more customers waiting, he works until the system implies then takes another vacation. If the server returns form a vacation and finds no customers waiting, he takes another vacation immediately. The M*/G/1 queue with vacation time has been studied by a number of workers such as Copper (1970), Levy and Yechiali (1975), Scholl and Kleinrock (1983). In particular Scholl, M. and Kleinrock (1983) has studied the waiting time of the M*/G/1 queue with a cation time under three queuing disciplines, which are independent of service time: First-come-first-served (FCFS), Service in random order (SIRO) and non-preemptive LCFS. Several authors have studied the batch arrival queueing system /G/1 under FCFS [Chaudhary (1979), Chaudhary and Templeton (1983), Gros and Harris (1985) and Kleinrock (1975)] Recently Baba (1986) has studied the M*/G/1 queue with vacation time under FCFS However, there has been no work for the M* /G/1 queue with vacation time under other service discipline.

Non-preemptive LCFS queueing discipline is applicable to many practical situations such as pushdown stack and inventory system, etc. The batch arrival queueing model appears in many situations such as computer commination systesm. So it is important to analyses the mode that will be studied in this chapter. In the batch arrival queueing system under non-preemptive LCFS, the following two queueing disciplines are considered:

1. QUEUEING DISCIPLINE 1:
The customers which are included in the same batch as the customer in service are served previously than the customers who arrived at the system after the customer in service. (This discipline in considered as the case that arrived batch is treated as a super customer).

2. QUEUEING DISCIPLINE 2:
The order of service is non-preemptive LCFS with respect to batches and the order of service in a batch is random (This discipline is considered as the non-preemptive LCFS discipline with respect to all customers in the system)
In the present chapter, we study the queueing system under discipline 2. This chapter provides Laplace stieljges transform (LST) of the distribution of function of the steady stage waiting time for the M*/G/1 queue with and without vacation time under non-preemptive LCFS. The first two moments of the waiting time are obtained.
The relationship that we find in he present work between the second moment of the waiting time for the FCFS and LCFS in M*/G/1 queue with and without vacation time is an extension of the result which has been found in the M/G/1 queue with and without vacation time.

ASSUMPTIONS AND NOTATIONS

We consider the M*/G/1 queue with and without vacation time. We study the queueing system under queueing discipline 2 stated in the previous section. For the M*/G/1 queue, it is assumed that customers arrive in batches according to a time homogenous Poisson process with rate. The batch size is a random variable and

$$P(X=n)=g_{n,} \qquad n=1,2\ldots\ldots \qquad \text{...(1)}$$

with probability generating function (*pgg*)

$$G(z)=\sum_{i=1}^{\infty} g_K Z^k \qquad |Z|\le 1 \qquad \text{...(2)}$$

We assume that n^{th}(n = 1, 2, 3) factorial moments of X are finite and defined by

$$g=E(X)=G^{(1)}(g) \qquad \text{...(3)}$$

$$g^{(2)}=E[X(X-1)]=G^{(2)}(1) \qquad \text{...(4)}$$

$$g^{(3)}=E[X(X-1)(X-2)]=G^{(3)}(1) \qquad \text{...(5)}$$

S = Service the time of a typical customer
S (*X*) = Distribution function of *S*
S*(θ) = LST of the *S*

The distribution function *S*(*x*) and LST (*S**(θ)) of *S* are given by

$$S(X)=\frac{1}{E(S)}\int_o^X [l-S(t)]dt \qquad \text{...(6)}$$

$$S^*(\theta)=\frac{[1-S^*(\theta)]}{\theta.E(S)} \qquad \text{...(7)}$$

In order to analyse the queue with vacation time, we denote
V = Typical vacation time
V(*X*) = Distribution function of *V*
*V**(θ) = LST of the *V*
The distribution function *V*(*x*) and LSTV*(θ) of *V* are given by

$$V(X)=\frac{1}{E(V)}\int_0^X[1-V(t)]dt \qquad ...(8)$$

$$V^*(\theta)=\frac{[1-V^*(\theta)]}{\theta.E(V)} \qquad ...(9)$$

Let-

B = Busy period that starts with one customer

$B(X)$ = Distribution function of B

$B^*(\theta)$= LST of the B

S_1 = Service time of one customer during one busy period (one let it is S_1=x)

N_1 = Number of customers arrivals during the interval S_1

By conditioning on S_1 and N_1, we have the following conditional transform

$$E(e^{-\theta B}/S_1=X,N_1=n)=e^{-\theta X}[B^*(\theta)]^X \qquad ...(10)$$

Unconditioning on we have-

$$E(e^{-\theta B}/S_1=X)=\sum_{n=0}^{\infty}E(e^{-\theta B}S_1=X,N_1=n)\,P(N_1=n)$$

$$=\sum_{n=0}^{\infty}e^{-\theta X}[B^*(\theta)]^X\sum_{k=0}^{\infty}\frac{(\lambda X)^k}{k}g_n^k e^{-kX}$$

$$=\exp[-\{(\theta-\lambda-\lambda G)(B^*(\theta)\}]X \qquad ...(11)$$

where g_n^{-k} is the k-fold convolution of g_n with itself, with $g_n^k=\delta_{10}$. Finally, we have

$$B^*(\theta)=E(e^{-\theta B})$$

$$=\int_0^{\infty}\exp[-\{\theta+\lambda-\lambda G(B^*(\theta))\}X]dS(X)$$

$$=S^*(\theta+\lambda-\lambda G(B^*(\theta))) \qquad ...(12)$$

In the case of the traffic $\rho=\lambda g(E(S)<1$, intensity n^{th} $(n=1,2,3$) moments of $B(X)$ are given by

$$E(B)=\frac{E(S)}{(1-\rho)} \qquad ...(13)$$

$$E(B^2)=\frac{E(S^2)+\lambda g^{(2)}[E(S)]^3}{(1-\rho)^3} \qquad ...(14)$$

$$E(B^3)=\frac{E(S^3)+\lambda g^{(3)}[E(S)]^4}{(1-\rho)^4}+\frac{3\lambda g[E(S^2)]^2+6\lambda g^{(2)}[E(S^2)]E(S)]^2+3\lambda[g^{(2)}]^2[E(S)]^3}{(1-\rho)^5} \qquad ...(15)$$

WAITING TIME DISTRIBUTION

1. The M*/G/1 QUEUE WITHOUT VACATION TIME UNDER LCFS:

Let r_n $(n=1,2...)$ be the probability of an arbitrary customer being in the n^{th} position of an arrived batch. Burke (1975) showed, using a result in the renewal theory that it is given by

$$r_n=\frac{1}{g}\sum_{k=n}^{\infty}g_k \qquad ...(16)$$

Define the p.g.f.

$$R(z)=\sum_{n=1}^{\infty}r_nZ^n=\frac{Z[1-G(Z)]}{g(1-Z)} \qquad ...(17)$$

At the time of an arbitrary test customer's arrival, there will be a number of customers arriving in his batch who will be served before him. This number is $(n-1)$ with probability r_n. Suppose that an arbitrary test customer named C_0 arrives when the server is idle. The (n-1) customers arriving in C_0's is batch who will be served before him are named C_1……..C_{n-1} respectively, Observe that the effect of C_i (i=1,2….. n-1) on the waiting time C_0 is the one busy period B_i generated by C_i since B_1……,B_{n-1} are mutually independent and the proportion of time that the server is idle in an $M^X/G/1$ queue is (1-ρ), the steady state waiting time C_0 of is distributed as B_1 + - - - + B_{n-1} with probability (1-ρ)r_n. Next suppose that C_0 arrives when the server is busy. Let the remaining service time of the customer in service is C_0 is arrival epoch be *S* Suppose that '*m*' (a random variable) customers named $C_n,\ldots\ldots,C_{m+n-1}$ arrive during *S*. Denote by $B_i(i=1,2,\ldots.,m+n-1)$ the one busy period generated by C_i, similar to the case that the server is idle. The steady state waiting time of C_0 is distributed as B_1+….+B_{n-1}+B_n+…+B_{m+n-1} with probability ρr_n.

Let,

W_L = Steady state waiting time of an arbitrary customer

$W_L^*(\theta)$ = LST of W_L

Since $B_1,\ldots\ldots,B_{n-1}$ are independent of S_1 we have

$$W_L^*(\theta)=\sum_{n=1}^{\infty}(1-\rho)r_n[B^*(\theta)]^{n-1}\sum_{n=1}^{\infty}\rho r_n\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}[B^*(\theta)]^{m+n-1}$$

$$\int_0^{\infty}\frac{(\lambda t)^k}{K!}e^{-(\lambda+\theta)t}g_m^*dS(t)$$

$$=\frac{(1-\rho)\sum_{n=1}^{\infty}[B^*(\theta)]^t}{[B^*(\theta)]}+\sum_{n=1}^{\infty}\rho r_n[B^*(\theta)]^{n-1}\int_0^{\infty}\left[\sum_{n=0}^{\infty}[B^*(\theta)]^m e^{-\theta t}\sum_{k=0}^{m}\frac{(\lambda t)^k}{K!}g_m^k e^{-\lambda t}\right]dS(t)$$

Using equations (17) and (11) we have

$$=\frac{(1-\rho)R(B^*(\theta)]}{[B^*(\theta)]}+\sum_{n=1}^{\infty}\rho r_n[B^*(\theta)][-\{\theta+\lambda-\lambda G(B^*\theta)\}t]dS(t)$$

Using equation (17) and (12) we have

$$=\frac{(1-\rho)[B^*(\theta)[1-G(B^*(\theta)]}{[B^*(\theta)g[1-B^*(\theta)]}+\sum_{n=1}^{\infty}\rho r_n[B^*(\theta)]^{n-1}S^*\{\theta+\lambda-\lambda G(B^*\theta))]$$

$$=\frac{(1-\rho)[1-G(B^*(\theta))]}{g[1-B^*(\theta)]}+\left[\sum_{n=1}^{\infty}r_n[B^*(\theta)]^n\right]\rho(B^*(\theta)]^{-1}S^*\{\theta+\lambda-\lambda G(B^*(\theta)\}]$$

Using Equation (17) and (7), we have

$$=\frac{(1-\rho)[1-G(B^*(\theta))]}{g[1-B^*(\theta)]}+\frac{B^*(\theta)[1-G(B^*(\theta))]}{g[1-B^*(\theta)]}\rho(B^*(\theta))^{-1}\frac{[1-S^*(\theta)]}{[\theta+\lambda-\lambda G(B^*(\theta))]E(S)}$$

$$=\frac{(1-\rho)[1-G(B^*(\theta))]}{g[1-B^*(\theta)]}+\frac{\rho[1-G(B^*(\theta))]}{g[1-B^*(\theta)]}\frac{[1-S^*(\theta)]\lambda g}{[\theta+\lambda-\lambda G(B^*(\theta))]}$$

$$W_L^*(\theta)=\frac{(1-\rho)[1-G(B^*(\theta))]}{g[1-B^*(\theta)]}+\frac{\lambda[1-G(B^*(\theta))]}{\{\theta+\lambda-\lambda G[(B^*(\theta)]\}}\quad\ldots.(18)$$

By taking the first and second derivatives of (18) at θ = 0 at we obtain the following expressions:

$$E(W_L)=\frac{(1-\rho)g^{(2)}E(B)}{2g}+\frac{\lambda g^{(2)}[E(B)]^2+\lambda gE(B)^2}{2[1+\lambda gE(B)]^2}\quad\ldots(19)$$

$$= \frac{\lambda g E(S^2)}{2(1-\rho)} + \frac{g^{(2)} E(S)}{2g(1-\rho)}$$

$$E(W_L^2) = \frac{(1-\rho)}{6g}[3g^{(2)}E(B)^2 + 2g^{(3)}E(B)^2] + \frac{\lambda}{6[1+\lambda g(E(B))]^3}[2(1+gE(B))][g^3[E(B)^3 + 3g^{(2)}E(B)E(B)^2] + g(B)^1]$$

$$-3\lambda[g^{(2)}\{E(B)\}^2 + gE(B^2)]^2]$$

$$= \frac{\lambda g(ES^3)}{3(1-\rho)} + \frac{\lambda g^{(2)}\{E(S)^2\}^2}{2(1-\rho)^3} + \frac{g^{(3)}\{E(S)\}^2}{3g(1-\rho)^2} + \frac{\lambda\{g^{(2)}\}^2\{E(S)\}^3 + (1+\rho)g^{(2)}E(S^2)}{2g(1-\rho)} \quad \text{...(20)}$$

REMARK 1:

Let W_F be the steady-state waiting time without vacation time when the service discipline is FCFS. Its LST, $W_F^*(\theta)$ is given by

$$W_F^*(\theta) = \frac{(1-\rho)\theta[1-G(S^*(\theta))]}{g[\theta-\lambda+\lambda G(S^*(\theta))][1-S^*(\theta)]} \quad \text{...(21)}$$

By taking the first and second derivatives of (21) at $\theta = 0$,

We have,

$$E(W_F) = \frac{\lambda g(E(S)^2)}{2(1-\rho)} + \frac{g^{(2)}E(S)}{2g(1-\rho)} \quad \text{...(22)}$$

$$E(W_F^2) = \frac{\lambda g E(S^3)}{3(1-\rho)} + \frac{\lambda^2 g^{(2)}\{E(S^2)\}^2}{2\,(1-\rho)^2} + \frac{g^{(3)}\{E(S)\}^2}{3(1-\rho)^2} + \frac{\lambda\{g^{(2)}\}^2\{E(S)\}^3 + (1+\rho)g^{(2)}E(S^2)}{2g(1-\rho)^2} \quad \text{...(23)}$$

Since the order of service is independent of time we see immediately that the mean queue size and the mean waiting time for LCFS must be same as for FCFS. Thus it is clear that $E(W_L) = E(W_F)$ from (19) and (22)

However, the second moment $E(W_L^2)$ is larger than $E(W_F^2)$. Comparing (20) and (23), we have-

$$E(W_F^2) = (1-\rho).E(W_L^2) \quad \text{...(24)}$$

It is surprising to find that this result holds for the M*/G/1 queue as well as for M/G/1.

2. The M*/G/1 QUEUE WITH VACATION TIME UNDER LCFS:

As in the case 1., since the proportion of time that the server is vacationing in the M*/G/1 queue with vacation time is (1-ρ), the steady-state waiting time of an arbitrary test customer is distributed as $V+B_1+....+B_{m+n-1}$ with probability $(1-\rho)r_n$, where *m* is the number of customers arrived during the residual vacating time *V* and as $S+B_1+...+B_{m+n-1}$ with probability ρr_n where *m* is the customers arrived during the residual service time *S*.

Let-

W_{VL} = Steady state waiting time of an arbitrary customer

$W_{VL}^*(\theta)$ = LST of W_{VL}

By using (7) and (17) we have-

$$W_{VL}^*(\theta) = \sum_{n=1}^{\infty}(1-\rho)r_n \sum_{m=0}^{\infty}\sum_{k=0}^{m}[B^*(\theta)^{m+n-1}]\int_0^{\infty}\frac{(\lambda t)^k}{k!}e^{(\lambda+\theta)t}g_m^k(dV(t))$$

$$+\sum_{n=1}^{\infty}\rho r_n \sum_{m=0}^{\infty}\sum_{k=0}^{m}[B^*(\theta)]^{m+n-1}\int_0^{\infty}\frac{(\lambda)t^k}{k!}e^{(\lambda+\theta)t}g_m^k dS(t)$$

$$= \sum_{n=1}^{\infty}(1-\rho)r_n[B^*(\theta)]^{n-1}V^*\{\theta+\lambda-\lambda G(B^*(\theta))\} + \sum_{n=1}^{\infty}\rho r_n[B^*(\theta)]^{n-1}S^*\{\theta+\lambda-\lambda G(B^*(\theta))\}$$

$$= \frac{(1-\rho)[1-V^*\{\theta+\lambda-\lambda G(B^*(\theta))\}R[B^*(\theta)]}{B^*(\theta)\{\theta+\lambda-\lambda G(B^*(\theta))\}E(V)}$$

$$= \frac{\rho[1-S^*\{\theta+\lambda-\lambda G(B^*(\theta))R \mid B^*(\theta)}{B^*(\theta)\{\theta+\lambda-\lambda G(B^*(\theta))\}E(S)}$$

$$W_{VL}^*(\theta) = \frac{(1-\rho)[1-V^*\{\theta+\lambda-\lambda G(B^*(B))\}][1-G(B^*(\theta))]}{g[1-B^*(\theta)]\{\theta+\lambda-\lambda G(B^*(B))\}E(V)} + \frac{\lambda[1-G(B^*(\theta))]}{\{\theta-\lambda-\lambda G(B^*(B))\}} \qquad ...(25)$$

By taking the first and second derivatives of (25) at $\theta = 0$,
We have-

$$E(W_{VF}) = \frac{E(V^2)}{2E(V)} + \frac{\lambda g E(S^2)}{2(1-\rho)} + \frac{g^{(2)}E(S)}{2g(1-\rho)} \qquad ...(26)$$

$$E(W_{VL}^2) = (1-\rho)E(W_{VF}^2) \quad \frac{E(V^3)}{(1-\rho)E(V)} + \frac{\lambda g E(S^2)E(V^2)}{2(1-\rho)^2E(V)} + \frac{\lambda g E(S^3)}{3(1-\rho)^2} + \frac{\lambda^2 g^{(2)}\{E(S^2)\}^2}{2(1-\rho)} + \frac{g^{(3)}\{E(S)\}^2}{3g(1-\rho)^2}$$
$$+ \frac{\lambda(g^2)^2[E(S)]^3 + (1+\rho)g^{(2)}E(S^2)}{2g(1-\rho)^2} \qquad ...(27)$$

REMARK 2:

Let W_{VE} be the steady state waiting time with vacation time when the service discipline is FCFS. Its LST, $W_{VL}^*(\theta)$ is given by

$$W_{VF}^*(\theta) = \frac{(1-\rho)[1-G(S^*(\theta))][1-V^*(\theta)]}{g[\theta-\lambda+\lambda G(S^*(\theta))][1-S^*(\theta)E(V)]} \qquad ...(28)$$

By taking the first and second derivatives of (28) at $\theta = 0$, We have

$$E(W_{VF}) = \frac{E(V^2)}{2E(V)} + \frac{\lambda g E(S^2)}{2(1-\rho)} + \frac{g^{(2)}E(S)}{2g(1-\rho)} \qquad ...(29)$$

$$E(W_{VF}^2) = \frac{g^{(2)}E(S)E(V^2)}{2g(1-\rho(E(V))} + \frac{E(V^3)}{3E(V)} + \frac{\lambda g E(S^2)E(V^2)}{2(1-\rho)E(V)}$$
$$+ \frac{\lambda g E(S^3)}{3(1-\rho)} + \frac{\lambda^2 g^{(2)}\{E(S^2)\}^2}{2(1-\rho)} + \frac{g^{(3)}\{E(S)\}^2}{3g(1-\rho)} + \frac{\lambda(g^{(2)})^2[E(S)]^3 + (1+\rho)g^{(2)}E(S^2)}{2g(1-\rho)^2} \qquad ...(30)$$

As in case 1, it can be stated that –

$$E(W_{VL}) = E(W_{VF})$$

and

$$E(W_{VL}^2) = (1-\rho)E(W_{VL}^2)$$

CONCLUSION

We have studied the waiting time of the M*/G/1 queue without and with vacation time under non-preemptive LCFS Comparing it with FCFS, have seen that For the M*/G/1 queue, equations (31)-(34) hold (School and Kleinrock, 1983)-

$$E(W_L) = E(W_F) \qquad ...(31)$$

$$E(W_F^2) = (1-\rho)E(W_L^2) \qquad ...(32)$$

$$E(W_{VL}) = E(W_{VF}) \qquad ...(33)$$

$$E(W_{VF}^2) = (1-\rho)E(W_{VL}^2) \qquad ...(34)$$

For the M/G/1 queue, equations (31)-(34) hold [Scholl and Kleinrock. (1983)]

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