



ORIGINAL ARTICLE

Stoke's Flow Problem through Porous Medium in Presence of Magnetic Field with Transverse Periodic Suction

Narendra Singh

Department of Mathematics,

Rani Shanti Devi Mahavidyalaya, Hathaundha, Ram Sanahi Ghat, Barabanki

Email: nrnarendrarajput@gmail.com

ABSTRACT

In this section we study of an approximate solution to a three dimensional free convection flow of a viscous, incompressible fluid past an impulsively started infinite, vertical porous limiting surface through porous medium with transverse sinusoidal suction in presence of uniform magnetic field. Using perturbation technique, the expressions for the transient velocity in the main flow direction, the temperature, the sinusoidal skin fraction and the rate of heat transfer have been discussed with their respective dependence on the Prandtl number Pr , Grashoff number Gr , magnetic parameter M and porosity parameter K .

Key words: Magnetic field, porous medium, free convection, stoke's problem

Received: 9th July 2017, Revised: 22nd August 2017, Accepted: 24th August 2017

©2017 Council of Research & Sustainable Development, India

How to cite this article:

Singh N. (2017): Stoke's Flow Problem through Porous Medium in Presence of Magnetic Field with Transverse Periodic Suction. *Annals of Natural Sciences*, Vol. 3[3]: September, 2017: 124-135.

INTRODUCTION

The phenomenon of MHD free convection has many importances in technological applications, e.g.- in cooling reactors, providing heat sinks in turbine blades etc. The effects of free stream oscillations on the boundary layer flow of a viscous fluid are also often encountered in engineering applications e.g., in the aerodynamics of a helicopter rotor or in a variety of bio-engineering problems, such as fluttering airfoil etc. Such a study was initiated by Lighthill (1954). By assuming the oscillatory flow to be super imposed on the steady non zero mean flow, he linearized the momentum equations and solved them by the integral method, Stuart (1955). Further extended this idea to study a two dimensional flow past an infinite porous plate. Further, Soundalgekar (1973a&b), and Soundalgekar and Pop (1974) analyzed the unsteady flow past an infinite vertical porous plate with constant and variable suction respectively. In all the study mentioned the plate is assumed to be stationary.

The flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate in its own plane was first studied by Stokes (1851), also known as Rayleigh's problem. Recently, Georgantopoulos (1979) has discussed the free convection effects on the oscillatory flow in the Stoke's problem past an infinite, vertical porous limiting surface with constant suction. Singh (1989) analyzed three dimensional oscillatory flows past a plate. Most of the authors have assumed the suction velocity either constant or variable with time, thus restricting themselves to the two dimensional flows only. One possible suction distribution is a transverse sinusoidal one which give rise to a cross flow and hence to a three-dimensional flow over the surface. Gorla and Singh (2005) have been discussed free convection effects on stoke's problem with transverse periodic suction.

Therefore, in this section we study of the free convection effects on flow of a viscous, incompressible fluid past an impulsively started infinite, vertical porous limiting surface through porous medium with transverse sinusoidal suction in presence of uniform magnetic field.

MATHEMATICAL FORMULATION

We consider three-dimensional free convection flow of a viscous, incompressible fluid past an impulsively started infinite, vertical porous limiting surface through porous medium with transverse sinusoidal suction in presence of uniform magnetic field, which consists of a basic uniform distribution superimposed with a sinusoidal distribution

$\varepsilon v_0 \cos \pi \frac{v_0 z'}{v}$. A coordinate system is assumed with limiting surface lying vertically on

$x' - z'$ plane such that the x' - axis is oriented in the direction of the buoyancy force and y' - axis is perpendicular to the plane of the limiting surface. Initially the limiting surface is at rest but at $t > 0$ it starts moving impulsively in its own plane with constant velocity U_0 and its temperature is instantaneously raised or lowered to T'_w which is thereafter maintained constant. Then under the usual Boussinesq's approximation the non-dimensional equations governing the problem are given by.

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \dots(1)$$

$$\frac{1}{4} \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{1}{4} \frac{\partial U}{\partial t} + Gr\theta + \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \left(M + \frac{1}{K} \right) (u - U_0) \quad \dots(2)$$

$$\frac{1}{4} \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \left(M + \frac{1}{K} \right) v \quad \dots(3)$$

$$\frac{1}{4} \frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \left(M + \frac{1}{K} \right) w \quad \dots(4)$$

$$\frac{1}{4} \frac{\partial \theta}{\partial t} + v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) \quad \dots(5)$$

The corresponding boundary conditions are given by-

$$\left. \begin{aligned} y=0: \quad u=1, \quad v=-(1+\varepsilon \cos \pi z), \quad w=0, \quad \theta=-1 \\ y \rightarrow \infty: \quad u=U(t), \quad v=-1, \quad w=0, \quad p=p_\infty, \quad \theta=0 \end{aligned} \right\} \quad \dots(6)$$

When the amplitude $\varepsilon \ll 1$, we assume the solution in the neighborhood of the limiting surface of the form

$$\left. \begin{aligned}
 u &= u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots \\
 v &= v_0 + \varepsilon v_1 + \varepsilon^2 v_2 + \dots \\
 w &= w_0 + \varepsilon w_1 + \varepsilon^2 w_2 + \dots \\
 p &= p_0 + \varepsilon p_1 + \varepsilon^2 p_2 + \dots \\
 \theta &= \theta_0 + \varepsilon \theta_1 + \varepsilon^2 \theta_2 + \dots
 \end{aligned} \right\} \dots(7)$$

and for the free stream-

$$U = 1 + \varepsilon e^{i\omega t} \dots(8)$$

Substituting (7) and (8) in equations (1) to (5) and comparing the coefficient of like power of ε , and neglecting those of ε^2 . The terms free from ε given below describe a steady two dimensional problem with constant suction at the limiting surface-

$$v'_0 = 0 \dots(9)$$

$$u''_0 - v_0 u'_0 - n u_0 = -n - Gr \theta_0 \dots(10)$$

$$v''_0 - v_0 v'_0 - n v_0 = p'_0 \dots(11)$$

$$w''_0 - v_0 w'_0 - n w_0 = 0 \dots(12)$$

$$\theta''_0 - Pr v_0 \theta'_0 = 0 \dots(13)$$

Where primes denote differentiation with respect to y . The corresponding boundary conditions are:

$$\left. \begin{aligned}
 y=0: \quad u_0 &= 1, \quad v_0 = -1, \quad w_0 = 0, \quad \theta_0 = 1 \\
 y \rightarrow \infty: \quad u_0 &= 1, \quad v_0 = -1, \quad w_0 = 0, \quad p_0 = p_\infty, \quad \theta_0 = 0
 \end{aligned} \right\} \dots(14)$$

The solution of equations (9) to (13) under the boundary conditions (14) are:

$$u_0 = 1 + \beta_3 \left[e^{-\beta_1 y} - e^{-Pr y} \right] \dots(15)$$

$$\theta_0 = e^{-Pr y} \dots(16)$$

With transverse velocity components $v_0 = -1$, $w_0 = 0$ and the pressure $p_0 = p_\infty$. Taking into account the solutions of the transverse velocity components of the above two dimensional problem, the terms the coefficient of ε give following equations:

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \dots(17)$$

$$\frac{1}{4} \frac{\partial u_1}{\partial t} + v_1 \frac{\partial u_0}{\partial y} - \frac{\partial u_1}{\partial y} = \frac{i\omega}{4} e^{i\omega t} + Gr\theta_1 + \left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \right) - n(u_1 - e^{i\omega t}) \quad \dots(18)$$

$$\frac{1}{4} \frac{\partial v_1}{\partial t} - \frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} \right) - nv_1 \quad \dots(19)$$

$$\frac{1}{4} \frac{\partial w_1}{\partial t} - \frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} \right) - nw_1 \quad \dots(20)$$

$$\frac{1}{4} \frac{\partial \theta_1}{\partial t} + v_1 \frac{\partial \theta_0}{\partial y} - \frac{\partial \theta_1}{\partial y} = \frac{1}{Pr} \left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2} \right) \quad \dots(21)$$

With boundary conditions:

$$\left. \begin{aligned} y=0: \quad u_1 &= 0, \quad v_1 = -\cos \pi z, \quad w_1 = 0, \quad \theta_1 = 0 \\ y \rightarrow \infty: \quad u_1 &= e^{i\omega t}, \quad v_1 = 0, \quad w_1 = 0, \quad p_1 = 0, \quad \theta_1 = 0 \end{aligned} \right\} \quad \dots(22)$$

Equations (17) to (21) describe the three dimensional flow. To solve these equations, we separate the variables y, z and t as follows:

$$\left. \begin{aligned} u_1 &= u_{11}(y)e^{i\omega t} + u_{12}(y)\cos \pi z \\ v_1 &= v_{11}(y)e^{i\omega t} + v_{12}(y)\cos \pi z \\ w_1 &= -\left\{ zv'_{11}(y)e^{i\omega t} + (1/\pi)v'_{12}(y)\sin \pi z \right\} \\ p_1 &= p_{11}(y)e^{i\omega t} + p_{12}(y)\cos \pi z \\ \theta_1 &= \theta_{11}(y)e^{i\omega t} + \theta_{12}(y)\cos \pi z \end{aligned} \right\} \quad \dots (23)$$

Expressions for v_1 and w_1 in equation (23) are chosen show that the equation of continuity (17) is satisfied.

Substituting (23) in equations (18) to (21) and equating harmonic terms, we get the following equations with corresponding boundary conditions:

$$u''_{11} + u'_{11} - \left(\frac{i\omega}{4} + n \right) u_{11} = -\frac{i\omega}{4} - Gr\theta_{11} + v_{11}u'_0 - n \quad \dots(24)$$

$$u''_{12} + u'_{12} - (\pi^2 + n)u_{12} = v_{12}u'_0 - Gr\theta_{12} \quad \dots(25)$$

$$\left. \begin{aligned} y=0: \quad u_{11} &= 0, \quad u_{12} = 0 \\ y \rightarrow \infty: \quad u_{11} &= 1, \quad u_{12} = 0 \end{aligned} \right\} \quad \dots(26)$$

$$v_{11}'' + v_{11}' - \left(\frac{i\omega}{4} + n \right) v_{11} = p_{11}' \quad \dots(27)$$

$$v_{11}''' + v_{11}'' - \left(\frac{i\omega}{4} + n \right) v_{11}' = 0 \quad \dots(28)$$

$$v_{12}'' + v_{12}' - (\pi^2 + n) v_{12} = p_{12}' \quad \dots(29)$$

$$v_{12}''' + v_{12}'' - (\pi^2 + n) v_{12}' = \pi^2 p_{12} \quad \dots(30)$$

$$\left. \begin{aligned} y=0: \quad v_{11} &= 0, \quad v_{12} = -1, \quad v_{12}'' = 0 \\ y \rightarrow \infty: \quad v_{11} &= 0, \quad v_{12} = 0, \quad p_{11} = 0, \quad p_{12} = 0 \end{aligned} \right\} \quad \dots(31)$$

$$\theta_{11}'' + \text{Pr} \theta_{11}' - \frac{i\omega}{4} \text{Pr} \theta_{11} = \text{Pr} v_{11} \theta_0' \quad \dots(32)$$

$$\theta_{12}'' + \text{Pr} \theta_{12}' - \pi^2 \theta_{12} = \text{Pr} v_{12} \theta_0' \quad \dots(33)$$

$$\left. \begin{aligned} y=0: \quad \theta_{11} &= 0, \quad \theta_{12} = 0 \\ y \rightarrow \infty: \quad \theta_{11} &= 0, \quad \theta_{12} = 0 \end{aligned} \right\} \quad \dots(34)$$

From these equations with the help of equation (23), the solutions for u_1 , v_1 , w_1 , p_1 and θ_1 are obtained as:

$$\begin{aligned} u_1 &= \left[1 - e^{-N_1 y} \right] e^{i\omega t} \\ &+ \left[\beta_{26} e^{-\beta_4 y} + \beta_{21} e^{-\beta_{17} y} - \beta_{22} e^{-\beta_{18} y} - \beta_{23} e^{-\beta_{19} y} + \beta_{24} e^{-\beta_{20} y} - \beta_{25} e^{-\beta_5 y} \right] \cos \pi z \end{aligned} \quad \dots (35)$$

$$v_1 = \left[n_1 n_2 e^{-\pi y} - n_4 e^{-\beta_4 y} \right] \cos \pi z \quad \dots(36)$$

$$w_1 = -\frac{1}{\pi} \left[\beta_4 n_4 e^{-\beta_4 y} - \pi n_1 n_2 e^{-\pi y} \right] \sin \pi z \quad \dots (37)$$

$$p_1 = n_2 e^{-\pi y} \cos \pi z \quad \dots (38)$$

$$\theta_1 = \left[\beta_9 e^{-\beta_5 y} - \beta_8 e^{-\beta_{17} y} + \beta_7 e^{-\beta_{18} y} \right] \cos \pi z \quad \dots (39)$$

Substituting (15), (16) and (35) to (39) in expressions for u and θ in equation (7), we get the expression for the main flow velocity and temperature field as:

$$u(y, z, t) = u_0(y) + \varepsilon u_1(y, z, t) \quad \dots (40)$$

$$\theta(y, z) = \theta_0(y) + \varepsilon \theta_1(y, z) \quad \dots (41)$$

The main flow velocity can now be expressed in terms of the unsteady fluctuating parts as follows:

$$u(y, z, t) = u_0(y) + \varepsilon [U_r \cos \omega t - U_i \sin \omega t + u_{12} \cos \pi z] \quad \dots (42)$$

Where,

$$U_r + iU_i = 1 - e^{-N_1 y} \quad \dots (43)$$

Hence, the expression for the transient velocity for $\omega t = \frac{\pi}{2}$ is given by:

$$u(y, z, \frac{\pi}{2\omega}) = u_0(y) + \varepsilon [u_{12} \cos \pi z - e^{-A_2 y} \sin B_2 y] \quad \dots (44)$$

From the velocity components u and w , we calculate the skin friction in the main flow direction and in the direction perpendicular to the main flow respectively in the non dimensional form as:

$$\tau_x = \frac{\tau'_x}{\rho U_0 v_0} = \left(\frac{\partial u}{\partial y} \right)_{y=0} = u'_0 + \varepsilon u'_1 \quad \text{at} \quad y = 0 \quad \dots (45)$$

$$\tau_z = \frac{\tau'_z}{\rho v_0^2} = \left(\frac{\partial w}{\partial y} \right)_{y=0} = -\frac{\varepsilon}{\pi} [\pi^2 n_1 n_2 - \beta_4^2 n_3] \sin \pi z \quad \dots (46)$$

In terms of the amplitude and the phase of skin friction equation (45) can be written as-

$$\tau_x = \tau_{zx} + \varepsilon |m| \cos(\omega t + \alpha) \quad \dots (47)$$

Where

$$\tau_{zx} = (\beta_{11} - \beta_{10}) + \varepsilon [\beta_{27} \cos \pi z - B_2]$$

$$m = m_r + im_i, \quad \tan \alpha = \frac{m_i}{m_r}$$

From the expression for the temperature field, we can calculate q , the rate of heat transfer as-

$$q = -\frac{q'v}{v_0 k(T' - T'_\infty)} = \left(\frac{\partial \theta}{\partial y} \right)_{y=0} = \theta'_0 + \varepsilon \theta_1 \quad \text{at} \quad y = 0$$

$$q = -Pr + \varepsilon \beta_{28} \cos \pi z$$

Where,

$$n = M + \frac{1}{K}$$

$$n_1 = \frac{\pi}{(\pi + n)}$$

$$n_2 = \frac{\beta_4^2}{n_1(\pi^2 - \beta_4^2)}$$

$$n_3 = (1 + n_1 n_2)$$

$$n_4 = \frac{\text{Pr } n_1 n_2}{\pi}$$

$$N_1 = A_2 + iB_2$$

$$N = (\pi^2 + n)$$

$$n_5 = \frac{\text{Pr}^2(n_1 n_2 - 1)}{\beta_4^2 + \text{Pr } \beta_4 - \pi^2}$$

$$\beta_1 = \frac{1 + \sqrt{1 + 4n}}{2}$$

$$\beta_2 = \text{Pr}^2 - \text{Pr} - n$$

$$\beta_3 = \frac{Gr}{\beta_2}$$

$$\beta_4 = \frac{1 + \sqrt{1 + 4N}}{2}$$

$$\beta_5 = \frac{\text{Pr} + \sqrt{\text{Pr}^2 + 4\pi^2}}{2}$$

$$\beta_6 = \beta_4^2 + \beta_4 \text{Pr} - \pi^2$$

$$\beta_7 = \frac{\text{Pr}^2 n_3}{\beta_6}$$

$$\beta_8 = \frac{\text{Pr } n_1 n_2}{\pi}$$

$$\beta_9 = \beta_8 - \beta_7$$

$$\beta_{10} = \beta_1 \beta_3$$

$$\beta_{11} = \beta_3 \text{Pr}$$

$$\beta_{12} = n_1 n_2 \beta_{11} + Gr \beta_8$$

$$\beta_{13} = n_4 \beta_{11} + Gr \beta_7$$

$$\beta_{14} = n_1 n_2 \beta_{10}$$

$$\beta_{15} = n_4 \beta_{10}$$

$$\beta_{16} = Gr \beta_9$$

$$\beta_{17} = \text{Pr} + \pi$$

$$\beta_{18} = \beta_4 + \text{Pr}$$

$$\beta_{19} = \pi + \beta_1$$

$$\beta_{20} = \beta_1 + \beta_4$$

$$\beta_{21} = \frac{\beta_{12}}{\beta_{17}^2 - \beta_{17} - N}$$

$$\beta_{22} = \frac{\beta_{13}}{\beta_{18}^2 - \beta_{18} - N}$$

$$\beta_{23} = \frac{\beta_{14}}{\beta_{19}^2 - \beta_{19} - N}$$

$$\beta_{24} = \frac{\beta_{15}}{\beta_{20}^2 - \beta_{20} - N}$$

$$\beta_{25} = \frac{\beta_{16}}{\beta_5^2 - \beta_5 - N}$$

$$\beta_{26} = \beta_{25} - \beta_{24} + \beta_{23} + \beta_{22} - \beta_{21}$$

$$\beta_{27} = \beta_{19} \beta_{23} - \beta_{20} \beta_{24} + \beta_{25} \beta_5 - \beta_{26} \beta_4 - \beta_{21} \beta_{17} + \beta_{22} \beta_{18}$$

$$A_1 = \left[\frac{(1+4n) + \sqrt{(1+4n)^2 + \omega^2}}{2} \right]^{\frac{1}{2}}$$

$$B_1 = \left[\frac{-(1+4n) + \sqrt{(1+4n)^2 + \omega^2}}{2} \right]^{\frac{1}{2}}$$

$$A_2 = \frac{1 + A_1}{2}$$

$$B_2 = \frac{B_1}{2}$$

RESULT AND DISCUSSION

In order to get a physical insight into the problem and for the purpose of discussing the results obtained, numerical calculations have been carried out for the transient velocity, the temperature, the rate of heat transfer and the sinusoidal skin-friction. In this section,

we study the effects of the Grashoff number Gr , Prandtl number Pr , magnetic parameter M and porosity parameter K .

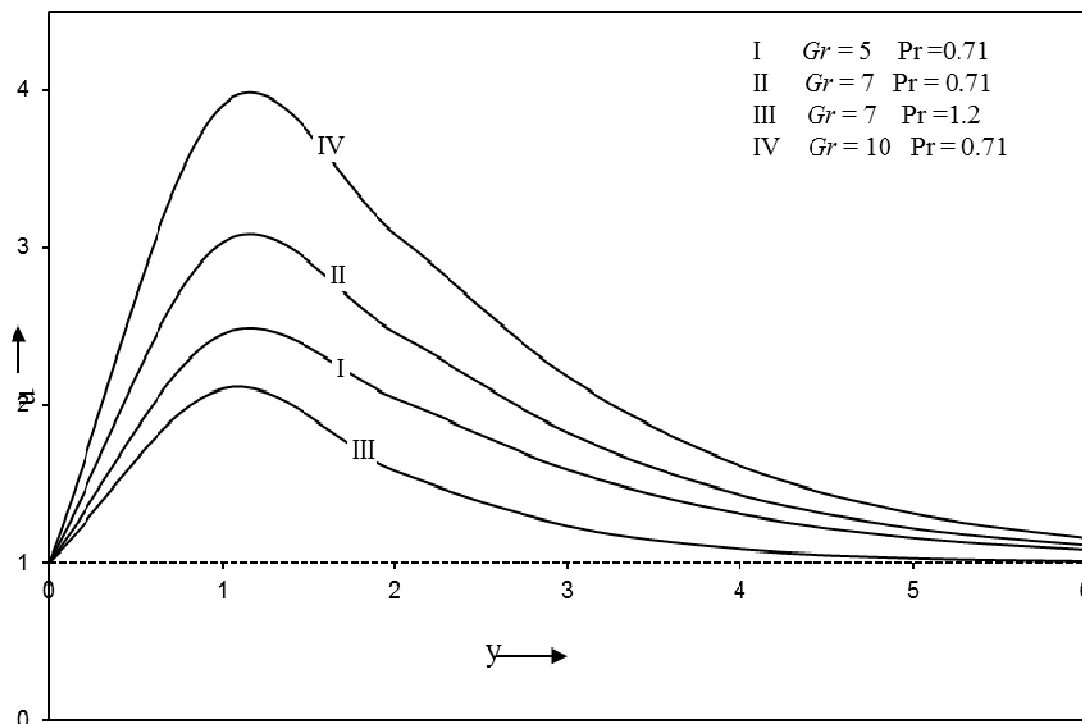


Fig. 1: Transient velocity profiles for different value of Pr and Gr .

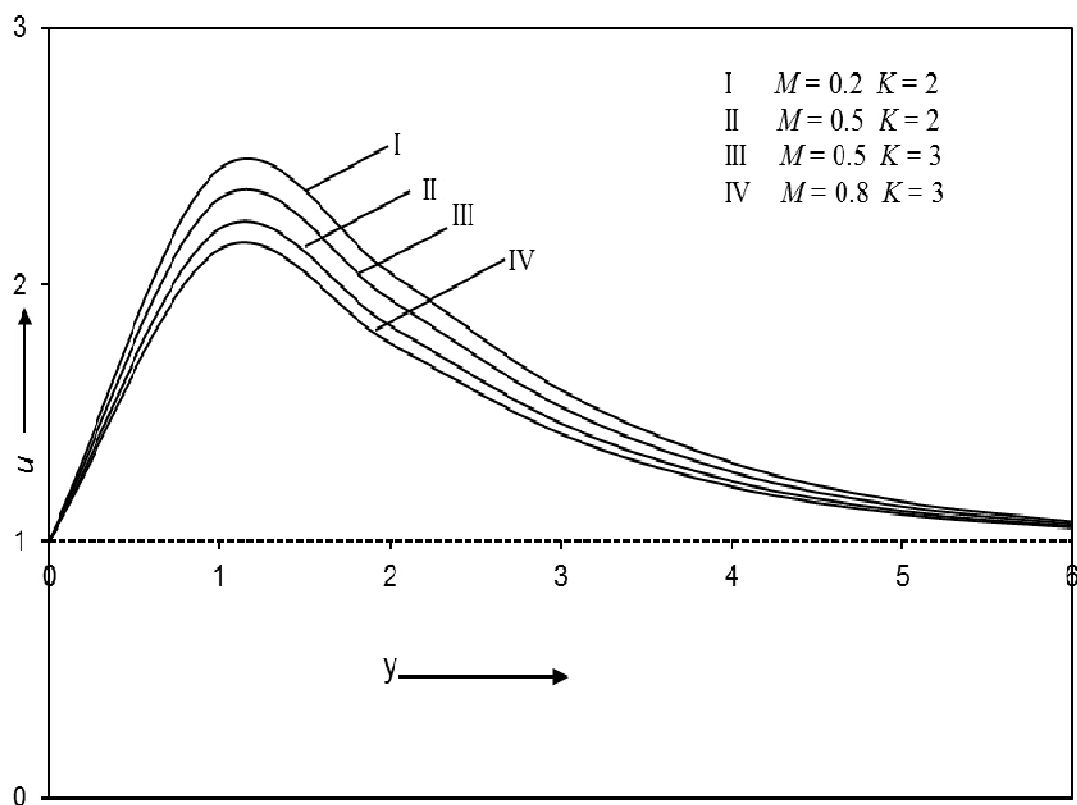


Fig. 2: Transient velocity for different value of M and K .

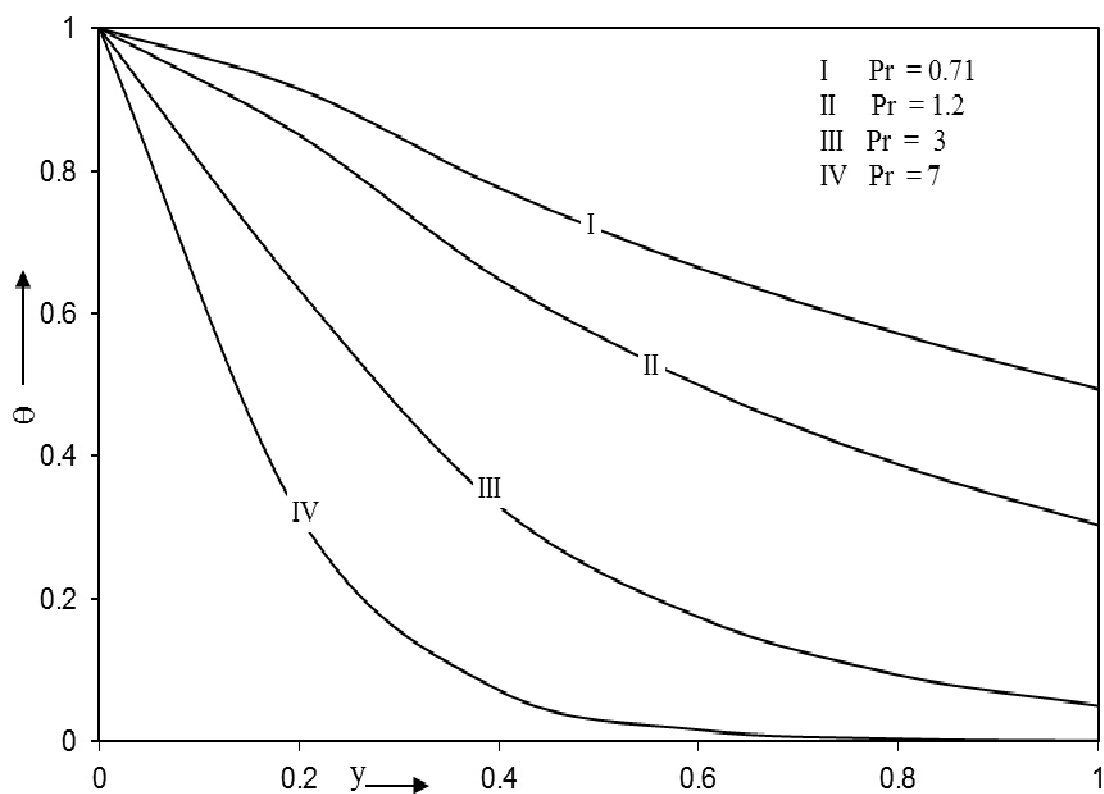


Fig. 3: Temperature profile for different value of Pr .

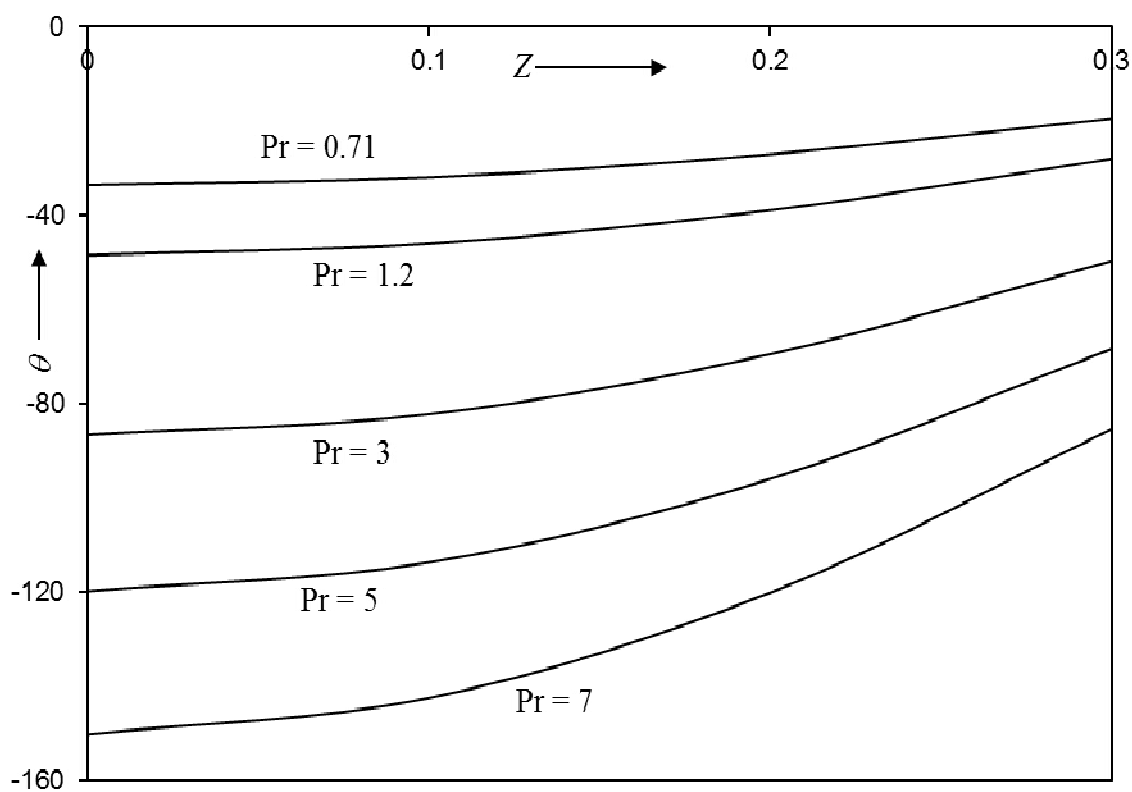


Fig. 4: Rate of heat transfer for different value of Pr .

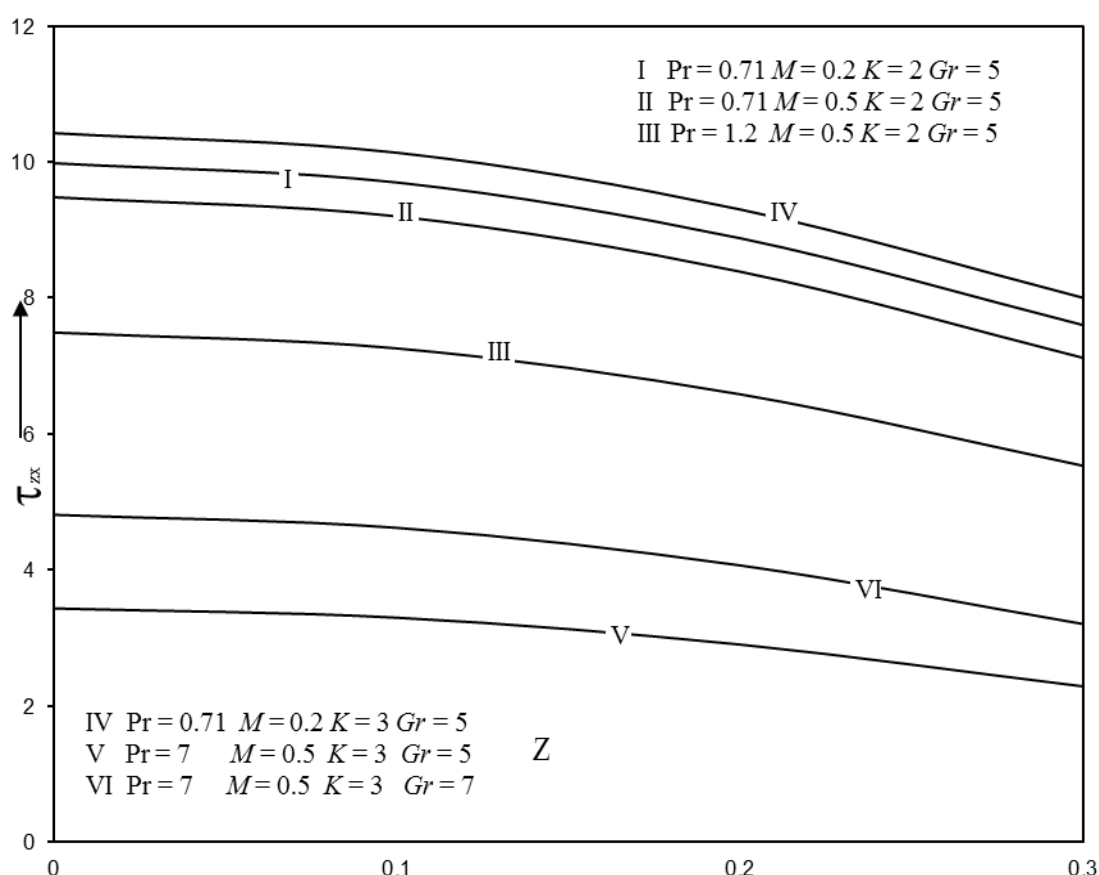


Fig. 5: Sinusoidal skin friction for different value of Pr , M , K and Gr .

The transient velocity profiles are plotted against y shown in the figure- 1, when the suction effects is maximum ($z = 0$) for $\omega = 5$, $\varepsilon = 0.2$, $M = 0.2$, and $K = 2$. It is observed that transient velocity increases sharply till $y = 1.2$, after it transient velocity decreases continuously with increasing in y . It is also observed that the transient velocity increases with increasing Grashoff number Gr but it decreases with increasing Prandtl number Pr .

The transient velocity profiles are plotted against y shown in the figure- 2, when the suction effects is maximum ($z = 0$) for $\omega = 5$, $\varepsilon = 0.2$, $Gr = 5$ and $Pr = 0.71$. It is observed that transient velocity increases sharply till $y = 1.2$, after it transient velocity decreases continuously with increasing in y . It is also observed that the transient velocity decreases with increasing magnetic parameter M but it increases with increasing porosity parameter K .

The temperature profiles are plotted against y shown in the figure- 3, when the suction effects is maximum ($z = 0$) for $\omega = 5$, $\varepsilon = 0.2$, $M = 0.2$, $K = 2$ and $Gr = 5$. It is observed that the temperature decreases with increasing Prandtl number Pr .

The rate of heat transfer is plotted against z shown in the figure- 4, for different value of Pr . It is observed that the rate of heat transfer decreases with increasing Prandtl number Pr .

The sinusoidal skin friction is plotted against z shown in the figure- 5, for different value of Pr , M , K and Gr . It is observed that the sinusoidal skin friction decreases with increasing Prandtl number Pr and magnetic parameter M , but it increases with increasing Grashoff number Gr and porosity parameter K .

CONCLUSION

The theoretical solution of three dimensional MHD free convection flow of a viscous, incompressible fluid past an impulsively started infinite, vertical porous limiting surface through porous medium with transverse sinusoidal suction. The study concludes the following results.

1. The transient velocity increases with increasing Grashoff number Gr but it decreases with increasing Prandtl number Pr .
2. The transient velocity decreases with increasing magnetic parameter M but it increases with increasing porosity parameter K .
3. The temperature decreases with increasing Prandtl number Pr .
4. The rate of heat transfer decreases with increasing Prandtl number Pr .
5. The sinusoidal skin friction decreases with increasing Prandtl number Pr and magnetic parameter M , but it increases with increasing Grashoff number Gr and porosity parameter K .

REFERENCES

1. Georgantopoulos G.A. (1979): Free convection effects on the Stokes problem past an infinite porous vertical limiting surface with constant suction-I. *Astrophysics and space Sci.*, 63: 491.
2. Gorla M.G. and Singh K.D. (2005): Free convection effects on the Stokes problem with transverse periodic suction. *Ganita*, Vol. 56, No. I.
3. Lighthill M.J. (1954): The response of laminar skin friction and heat transfer to fluctuations in the stream velocity. *Proc. Roy. Soc. London*, 224-A, I.
4. Singh K.D. (1989): Three dimensional oscillatory flow past a plate. *J. Math. Phys. Sci.*, 23.
5. Soundalgekar V.M. (1973a): Free convection effects on the oscillatory flow past an infinite, vertical, porous plate with constant suction-I. *Proc. Roy. Soc. London*, 333-A, 25.
6. Soundalgekar V.M. (1973b): Free convection effects on the oscillatory flow past an infinite, vertical, porous plate with constant suction-II. *Proc. Roy. Soc. London*, 333-A, 25.
7. Soundalgekar V.M. and Pop I. (1974): Viscous dissipation effects on unsteady free convection flow past an infinite vertical porous plate with variable suction. *Int. J. Heat Mass Transfer*, 17: 85.
8. Stokes G.G. (1851): On the effects of the internal friction of fluids on the motion of pendulums, *Camber. Phil.*, IX, 8.
9. Stuart J.T. (1955): A solution of the Navier-Stokes and energy equations illustrating the response of skin friction and temperature of an infinite plate thermometer of fluctuations in the stream velocity. *Proc. Roy. Soc. London*, 231-A, 116.