



ORIGINAL ARTICLE

Unsteady Effect on MHD Free Convection and Mass Transfer Flow of Kuvshinshki Fluid through Porous Medium with Constant Suction and Constant Heat and Mass Flux

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ABSTRACT

An analysis of unsteady two dimensional free convection and mass transfer flow of a Kuvshinshki fluid through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in the presence of a uniform magnetic field is presented. The effects of G_c and A_1 with the time on the velocity, temperature, concentration distribution and skin friction are discussed with the help of tables and graph.

Key words: Convection and Mass Transfer, Kuvshinshki Fluid, Porous Medium, Constant Suction

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INTRODUCTION

The effect of variable permeability on combined free and forced convection in porous media was studied by Chandrasekhara and Namboodiri (1985). Later on mixed convection in porous media adjacent to a vertical uniform heat flux surface was studied by Joshi and Gebhart (1985). Heat and mass transfer in the porous medium was discussed by Bejan and Khair (1985). The above problem was studied in presence of buoyancy effect by Trevisan and Bejan (1985). Lai and Kulacki (1990) studied the effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium. The free convection effect on the flow of an ordinary viscous fluid past and infinite vertical porous plate with constant suction and constant heat flux was investigated by Sharma (1991). The study of two dimensional flows through a porous medium bounded by a vertical infinite surface with constant suction velocity and constant heat flux in presence of free convection current was studied by Sharma (1992). Convection in a porous medium with inclined temperature gradient was investigated by Nield (1994). The problem of mixed convection along an isothermal vertical plate in porous medium with injection and suction was studied by Hooper, *et al.* (1994). Acharya, *et al.*, (2000) have discussed magnetic field effect on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Recently, Varshney and Kumar (2002) have studied the unsteady effect on MHD free convection and mass transfer flow through porous medium with constant suction and constant heat flux.

FORMULATION OF THE PROBLEM

We consider unsteady two dimensional motion of Kuvshinshki fluid through a porous medium occupying semi- infinite region of space normal to the direction of flow. The

effect of induced magnetic field is neglected. The Reynolds number is assumed to be small. Further magnetic field is not strong enough to cause Joule heating. Hence, the term due to electrical dissipation is neglected in energy equation (3). The X-axis is taken along the surface in the upward direction and Y-axis is taken normal to it. The fluid properties are assumed constant except for the influence of density in the body force term. As the bounding surface is infinite in length, all the variables are function of Y. Hence, by the Boundary layer approximation the basic equations for unsteady flow through porous medium are:

$$\frac{dV}{dY} = 0 \quad \dots (1)$$

$$\begin{aligned} \left(1 + \lambda_0 \frac{\delta}{\delta t_0}\right) \frac{\partial U}{\partial t_0} + V \frac{\partial U}{\partial Y} \\ = v \frac{\delta^2 U}{\delta Y^2} + g\beta(T - T_\infty) + g\beta'(C - C_\infty) \\ - \left(\sigma \frac{B_0^2}{\rho} + \frac{v}{K_0}\right) \left(1 + \lambda_0 \frac{\partial}{\partial t_0}\right) U \quad \dots (2) \end{aligned}$$

$$\frac{\partial T}{\partial t_0} + V \frac{\partial T}{\partial Y} = \frac{\lambda}{\rho C_p} \frac{\delta^2 T}{\delta Y^2} \quad \dots (3)$$

$$\frac{\partial C}{\partial t_0} + V \frac{\partial C}{\partial Y} - D \frac{\delta^2 C}{\delta Y^2} \quad \dots (4)$$

Where U and V are the corresponding velocity components along and perpendicular to the surface, p is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion, β' is the coefficient of concentration expansion, v is the Kinematic viscosity, T_∞ is the temperature of the fluid in the free stream, C_∞ is the concentration at infinite. σ is the electric conductivity, B_0 is the magnetic induction, k_0 is porosity parameter, λ is the thermal conductivity, D is the concentration diffusivity, C_p is the specific heat at constant pressure, λ_0 is the coefficient of viscoelastic.

METHOD OF SOLUTION

The equation of continuity (1) gives-

$$V = \text{constant} = -V_0 \quad \dots (5)$$

Where V_0 corresponds to steady suction velocity at the surface. In view of equation (5).

Equation (2), (3) and (4) can be written as-

$$\begin{aligned} \left(1 + \lambda_0 \frac{\delta}{\delta t_0}\right) \frac{\partial U}{\partial t_0} - V_0 \frac{\partial U}{\partial Y} \\ = v \frac{\delta^2 U}{\delta Y^2} + g\beta(T - T_\infty) + g\beta'(C - C_\infty) \\ - \left(\sigma \frac{B_0^2}{\rho} + \frac{v}{K_0}\right) \left(1 + \lambda_0 \frac{\partial}{\partial t_0}\right) U \quad \dots (6) \end{aligned}$$

$$\frac{\partial T}{\partial t_0} - V_0 \frac{\partial T}{\partial Y} = \frac{\lambda}{\rho C_p} \frac{\delta^2 T}{\delta Y^2} \quad \dots (7)$$

$$\frac{\partial C}{\partial t_0} - V_0 \frac{\partial C}{\partial Y} = D \frac{\delta^2 C}{\delta Y^2} \quad \dots (8)$$

The boundary condition of the problem is-

$$\left. \begin{aligned} u = 0, \quad dT/dY = q/\lambda, \quad dC/dY = -m/D \text{ at } Y = 0, \quad t_0 = 0 \\ U \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ as } Y \rightarrow \infty, \quad t_0 > 0 \end{aligned} \right] \quad \dots (9)$$

On introducing the following non dimensional quantities into equations (6), (7) and (8).

$$\begin{aligned} f(\eta) &= \frac{U}{V_0} (\text{velocity}), \quad \eta = \frac{V_0 Y}{\nu} (\text{Distance}), \quad \theta = \frac{(T-T_\infty)v_0 \lambda}{qv} \\ P_r &= \frac{\mu C_p}{\lambda} (\text{Prandtl number}), \quad S_c = \frac{\nu}{D} (\text{Schmidt number}) \\ \phi &= \frac{(C - C_\infty)v_0 D}{mv}, \quad t = \frac{t_0 V_0^2}{\nu}, \quad \alpha = \frac{K_0 V_0^2}{\nu^2} (\text{Porosity parameter}) \\ M &= \sigma \frac{B_0^2 \nu}{\rho V_0^2} (\text{Magnetic number}) \\ G_r &= g\beta \frac{qv^2}{V_0^4 \lambda} (\text{Grashoff number for heat transfer}) \\ G_m &= g\beta \frac{mv^2}{V_0^4 D} (\text{Grashoff number for mass transfer}) \end{aligned}$$

Where 'q' is the heat flux per unit area and 'm' is the mass flux per unit area.

$$\begin{aligned} -[1 + \lambda_1(M + \alpha^{-1})] \frac{\partial f}{\partial t} + \frac{\delta^2 f}{\delta \eta^2} + \frac{\partial f}{\partial \eta} - \lambda_1 \frac{\delta^2 f}{\delta t^2} - f(\alpha^{-1} + M) \\ = -G_r \theta - G_m \phi \end{aligned} \quad \dots (10)$$

$$-P_r \frac{\partial \theta}{\partial t} + \frac{\delta^2 \theta}{\delta \eta^2} + P_r \frac{\partial \theta}{\partial \eta} = 0 \quad \dots (11)$$

$$-S_c \frac{\partial \phi}{\partial t} + \frac{\delta^2 \phi}{\delta \eta^2} + S_c \frac{\partial \phi}{\partial \eta} = 0 \quad \dots (12)$$

Where viscoelastic parameter is-

$$\lambda_1 = \frac{\lambda_0 V_0^2}{\nu}$$

The corresponding boundary conditions become-

$$\left. \begin{aligned} \eta = 0, \quad f = 0, \quad \theta' = -1, \quad \phi' = -1 \\ \eta \rightarrow \infty, \quad f \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \end{aligned} \right] \quad \dots (13)$$

Following Mitra (1980), we assume the solution of

$$\left. \begin{aligned} f(\eta, t) &= f_0(\eta) e^{-\alpha t} \\ \theta(\eta, t) &= \theta_0(\eta) e^{-\alpha t} \\ \phi(\eta, t) &= \phi_0(\eta) e^{-\alpha t} \end{aligned} \right] \quad \dots (14)$$

Substituting equation (14) into equation (10), (11) and (12), we find-

$$\begin{aligned} f_0'' + f_0' - (\alpha^{-1} + M - n)(1 - n\lambda_1)f_0 \\ = -G_r \theta_0 - G_m \phi_0 \end{aligned} \quad \dots (15)$$

$$\theta_0'' + P_r \theta_0' + n P_r \theta_0 = 0 \quad \dots (16)$$

$$\phi_0'' + S_c \phi_0' + n S_c \phi_0 = 0 \quad \dots (17)$$

With corresponding boundary conditions

$$\left. \begin{aligned} f_0 = 0, \quad \theta_0' = -1, \phi_0' = -1 & \quad \text{at } \eta = 0 \\ f_0 \rightarrow 0, \quad \theta_0 \rightarrow 0, \quad \phi_0 \rightarrow 0, & \quad \text{as } n \rightarrow \infty \end{aligned} \right] \quad \dots (18)$$

Solving equations (15)-(17) under boundary conditions (18) we get-

$$f_0 = (A_4 G_r + A_5 G_m) e^{-A_4 \eta} - A_4 G_r e^{-A_2 \eta} - A_5 G_m e^{-A_3 \eta} \quad \dots (19)$$

$$\theta_0 = \frac{1}{A_2} e^{-A_2 \eta} \quad \dots (20)$$

$$\phi_0 = \frac{1}{A_3} e^{-A_3 \eta} \quad \dots (21)$$

Where

$$A_1 = \frac{1 + \sqrt[3]{1 + 4(1 - \eta \lambda_1)(\alpha^{-1} + M - n)}}{2}$$

$$A_2 = \frac{P_r + \sqrt[3]{P_r^2 - 4n P_r}}{2}$$

$$A_3 = \frac{S_c + \sqrt[3]{S_c^2 - 4n S_c}}{2}$$

$$A_4 = \frac{1}{A_2 [A_2^2 - A_2 - (\alpha^{-1} + M - n)(1 - n \lambda_1)]}$$

$$A_5 = \frac{1}{A_3 [A_3^2 - A_3 - (\alpha^{-1} + M - n)(1 - n \lambda_1)]}$$

Hence the equations for f , θ and ϕ will be as follows-

$$f = [(A_4 G_r + A_5 G_m) e^{-A_4 \eta} - A_4 G_r e^{-A_2 \eta} - A_5 G_m e^{-A_3 \eta}] e^{-nt} \quad \dots (22)$$

$$\theta = \frac{1}{A_2} e^{-A_2 \eta} e^{-nt} \quad \dots (23)$$

$$\phi = \frac{1}{A_3} e^{-A_3 \eta} e^{-nt} \quad \dots (24)$$

SKIN FRICTION

The skin friction coefficient at the surface is given by-

$$\tau = \left(\frac{\tau_{xy}}{\rho V_0^2} \right), \quad \eta=0 \quad \dots (25)$$

$$\tau = [A_4 G_r (A_2 - A_1) + A_5 G_m (A_3 - A_1)] \cdot e^{-nt} \quad \dots (26)$$

RESULTS AND DISCUSSION

Fluid Velocity Profiles of boundary layer flow are plotted in Fig. 1-3 for $F_r = 0.71$, $M=1.0, \alpha=1.0, S_c=0.6, n=0.1, G_{\eta_1}=2$ and different values G_r, λ_1 and t .

G_r, λ_1

For graph-I	5	2
For graph-II	10	2
For graph-III	5	4
For graph-IV	-5	4
For graph-V	-10	2
For graph-VI	-5	4

Variation in velocity for viscous fluid in unsteady flow is shown in tables from I to III and figs. From 1 to 3 and graphs from 1 to 3 for $G_r > 0$ and graphs from 4 to 6 for $G_r < 0$. These tables and Figs. and Graphs also illustrate the effect of $G_r > 0$, i.e., 5, 10 and $G_r < 0$, i.e., -5, -10 and λ_1 from 2 to 4 and time 0 to 4.

It velocity decreases continuously with the increase in η It is also concluded that the velocity decreases with the decreases in G_r and λ_1

By perusing graphs from 4 to 6 (below η -axis) of Fig. 1 separately for $t=0$ it is found that the velocity decreases gradually till $\eta = 1.2$ after it velocity increases gradually till $\eta = 3.5$ then after it velocity increases continuously with the increase in η . It is also concluded that the velocity decreases with the decrease in G_r from -5 to -10 and the increase in λ_1 from 2 to 4.

Comparing Graphs 1 to 3 (above η -axis) of Fig. 2 and 3 with graphs I to III (above η -axis) of fig. I It is noticed that each velocity Graph of Fig. 2 and 3 is higher than the respective graph of Fig. 1. The means that the fluid velocity increases with the increase in time t from 0 to 4.

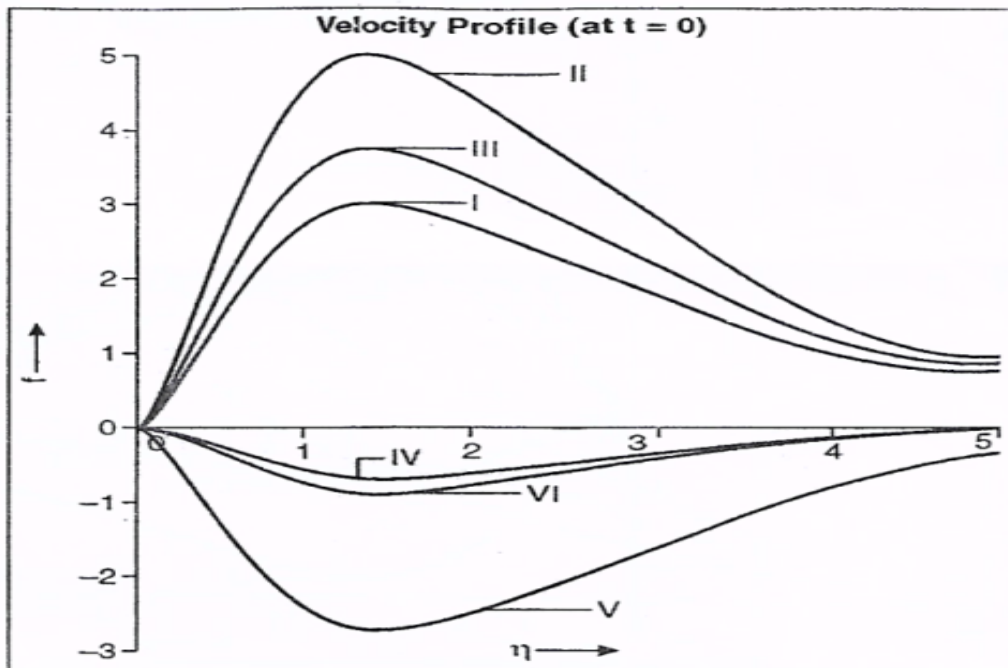


Fig. 1.

Fig. 1: Fluid velocity profile at $t=0$

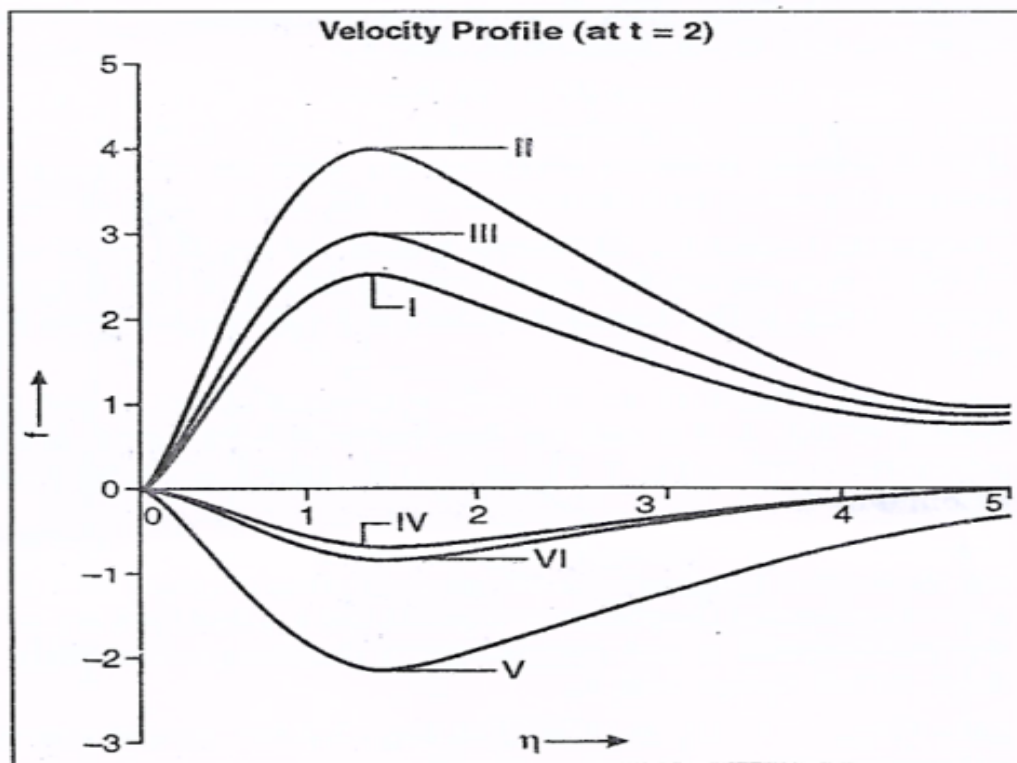


Fig. 2.

Fig. 2: Fluid velocity profile at t=2

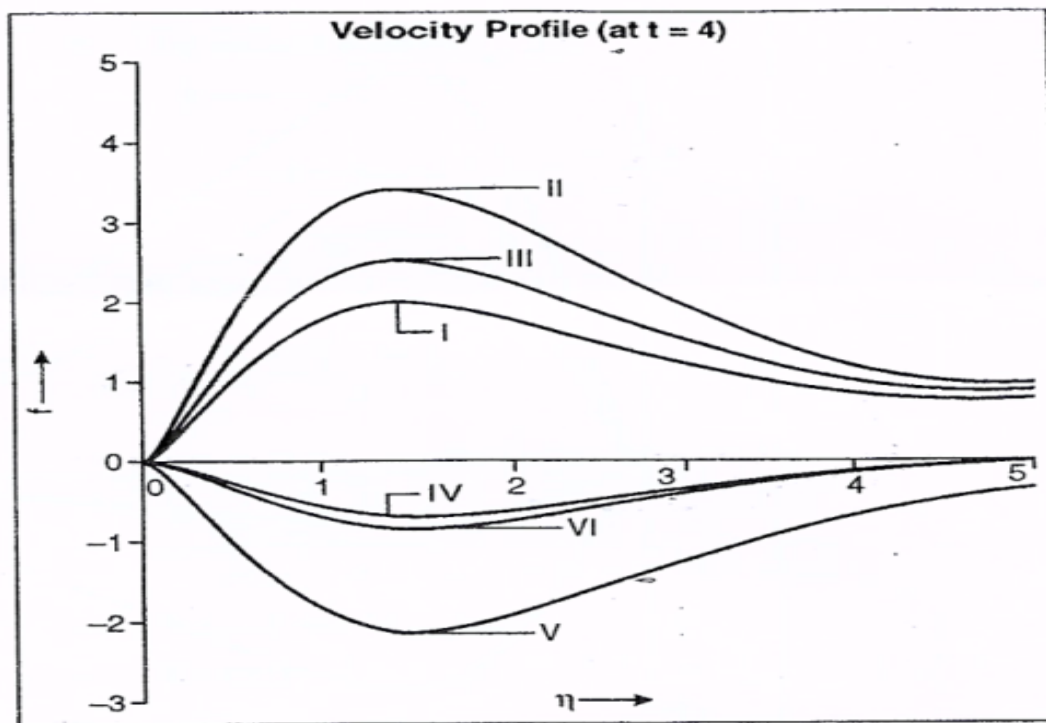


Fig. 3.

Fig. 2: Fluid velocity profile at t=4

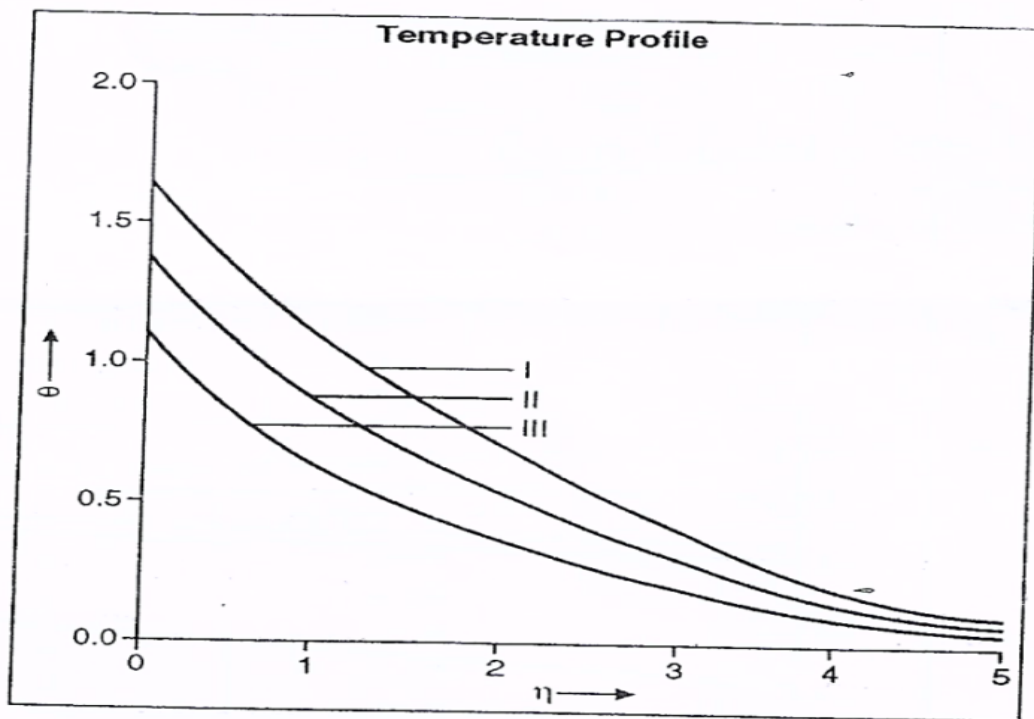


Fig. 4: Variation in viscosity of fluid with temperature

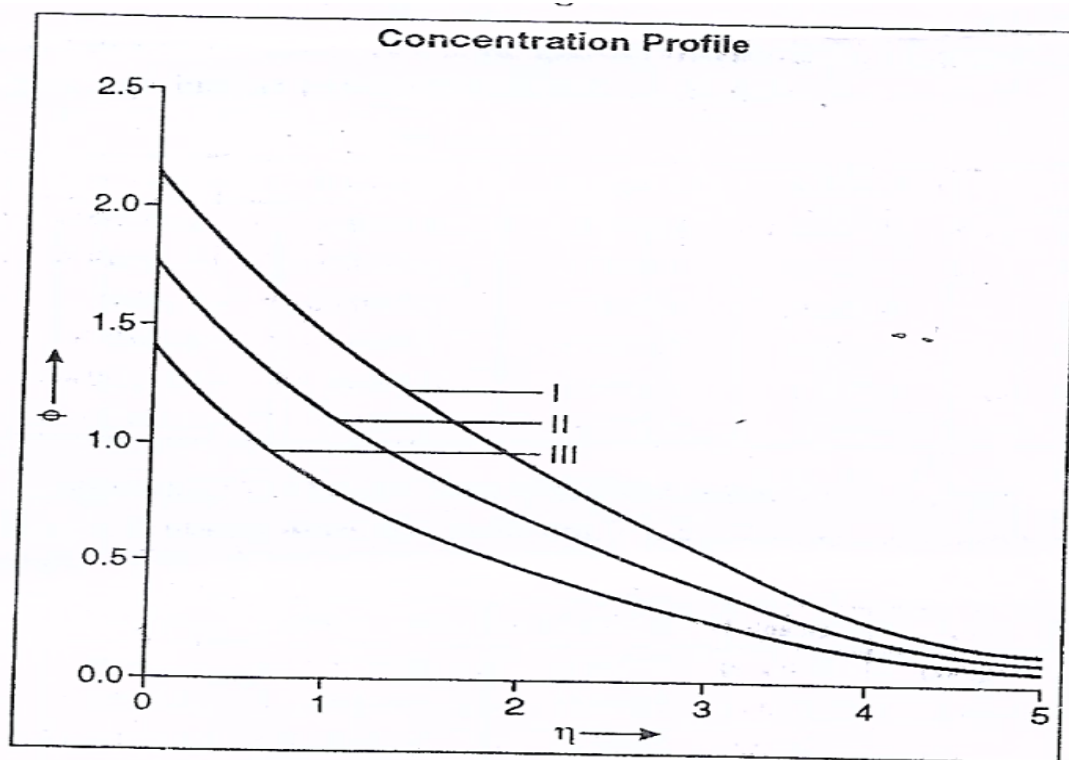


Fig. 5.

Fig. 5. Variation in viscosity of fluid with concentration

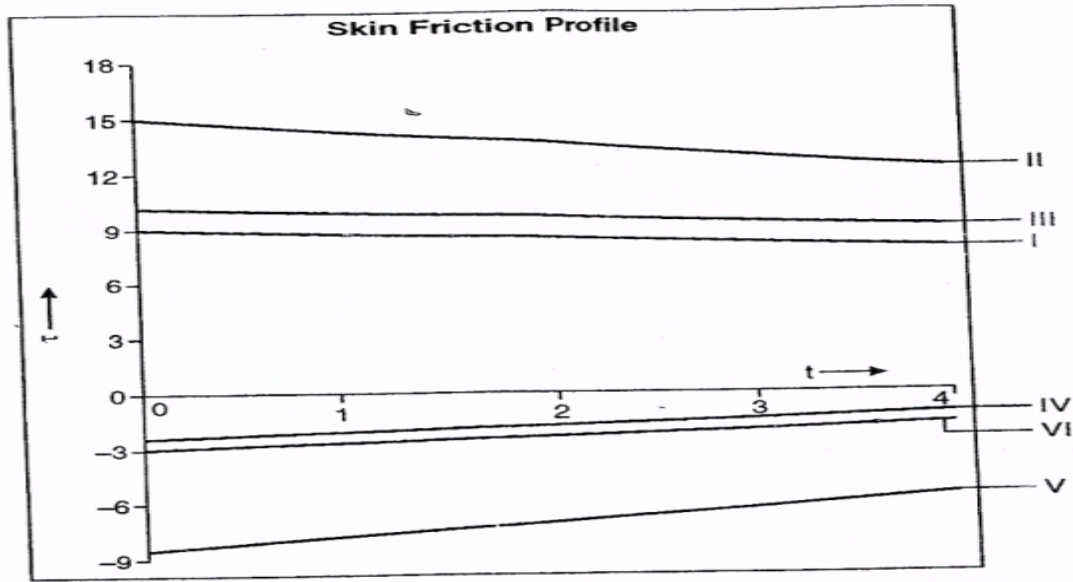


Fig. 6.

Fig. 6: Skin friction profile with viscosity

Table 1: Values of velocity at $t=0$, $P_r = 0.71$, $M=1.0$, $\alpha=1.0$, $S_e=0.6$, $n=0.1$, $G_m=2.0$ and different values of G_r and λ_1

η	Value of ' f ' at $t=0$					
	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V	Graph-VI
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1	2.97957	4.84971	3.55699	-0.76070	-2.63084	-0.89013
2	2.19772	3.52407	2.71664	-0.45498	-1.78133	-0.55281
3	1.34920	2.12523	1.69691	-0.20288	-0.97892	-0.25188
4	0.79642	1.23004	1.00936	-0.07081	-0.50443	-0.08895
5	0.46670	0.70591	-0.59330	-0.01173	-0.25094	-0.01483

Table 2: Values of velocity at $t=2$, $P_r = 0.71$, $M=1.0$, $\alpha=1.0$, $S_e=0.6$, $n=0.1$, $G_m=2.0$ and different values of G_r and λ_1

η	Value of ' f ' at $t=0$					
	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V	Graph-VI
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1	2.43947	3.97061	2.91221	-0.62281	-2.15395	-0.72877
2	1.79934	2.88526	2.22420	-0.37250	-1.45843	-0.45260
3	1.10463	1.73999	1.38931	-0.16611	-0.80147	-0.20622
4	0.65206	1.00707	0.82639	-0.05798	-0.41300	-0.07282
5	0.38210	0.57795	0.48576	-0.00960	-0.20545	-0.01214

Table 3: Values of velocity at $t=2$, $P_r = 0.71$, $M=1.0$, $\alpha=1.0$, $S_e=0.6$, $n=0.1$, $G_m=2.0$ and different values of G_r and λ_1

η	Value of ' f ' at $t=0$					
	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V	Graph-VI
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1	1.99727	3.25086	2.38432	-0.50991	-1.76350	-0.59667
2	1.47317	2.36225	1.82102	-0.30498	-1.19406	-0.37056
3	0.90439	1.42459	1.13747	-0.13600	-0.65619	-0.16884
4	0.53386	0.82452	0.67659	-0.04747	-0.33813	-0.05962
5	0.31284	0.47319	0.39770	-0.00786	-0.16821	-0.00994

Table 4: Values of temperature at $P_r=0.71, n=0.1$ and different values of t

η	Graph-I at $t=0$	Graph-II at $t=2$	Graph-III at $t=4$
0	1.6667	1.3646	1.1172
1	0.9147	0.7489	0.6131
2	0.5020	0.4110	0.3365
3	0.2755	0.2256	0.1847
4	0.1512	0.1238	0.1014
5	0.0830	0.0679	0.0556

Table 5: Values of concentration at $S_c=0.6, n=0.1$ and different values of t

η	Graph-I at $t=0$	Graph-II at $t=2$	Graph-III at $t=4$
0	2.1277	1.7420	1.4262
1	1.3298	1.0887	0.8914
2	0.8311	0.6805	0.5571
3	0.5195	0.4253	0.3482
4	0.3247	0.2658	0.2176
5	0.2029	0.1661	0.1360

Table 6: skin friction at $P_r=0.71, M=1.0, \alpha=1.0, S_c=0.6, n=0.1, G_m=2.0$ and different values of G_r and λ_1

η	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V	Graph-VI
0	9.15168	15.05568	10.32040	-2.65632	-8.56032	-2.96360
2	7.49276	12.32655	8.44963	-2.17481	-7.00860	-2.42639
4	6.13455	10.09212	6.91797	-1.78058	-5.73815	-1.98656

Comparing Graphs 4 to 6 (below η -axis) of Fig. 2 and 3 with Graph 4 to 6 (below η -axis) of fig. I. It is noticed that each velocity graph of fig. II and III is higher than the respective Graph of Fig. 1. The means that the fluid velocity with the increase in time t from 0 to 4. Temperature and Concentration Profiles are plotted in Fig. IV and V. From these figures it is concluded that temperature and concentration decrease with the increase in η . The temperature and concentration graphs at $t=0$ are at the top and $t=4$ are at the bottom. Thus, it is concluded that the temperature and concentration decrease considerably with the increase in time t . but these do not change with λ_1 . Skin friction profile is plotted in figure 6 for the different values of G_r and λ_1 as taken for velocity profile. By perusing graph- 1, 2 and 3 (above t -axis) of figure 6 it is observed that the skin friction decreases gradually with the increase in time t . comparing graph II and III with I it is obvious that skin friction increases with the increase in G_r and λ_1 .

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