e-ISSN: 2455-667X

Annals of Natural Sciences
Vol. 3(1), March 2017: 38-47
Journal's URL: http://www.crsdindia.com/ans.html
Email: crsdindia@gmail.com

Annals of Natural Sciences

ORIGINAL ARTICLE

Stochastic Analysis of Two Unit Cold Standby System with Three Levels of Priority Unit

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ABSTRACT

The present study deals with a system comprises of two non-identical units in parallel configuration. Initially first unit is operative and the second is kept as cold standby. The first unit gets priority in operation as well as in repair while which the second unit is treated as ordinary. The first unit starts operation from higher level and then passes through middle and lower level before reaching to failure stage. Similarly, if the charging process is going on then an operative unit reaches from lower to middle and from middle to high level. The priority unit can reach to failure stage only from middle level. Probability that an operative unit will be in higher, middle and lower level of stages are fixed. A single repair facility is available in the system to repair the failed priority and ordinary unit. The failure time distributions of priority and ordinary units are exponential. Also, the rate of transition form higher to middle, middle to lower, lower to middle and middle to higher are exponential with different parameters. The repair time distributions for priority and ordinary units are general.

***Key words**: Stochastic Analysis, Unit Cold Standby System, Priority Unit*

Received: 24th December 2016, Revised: 25th February 2017, Accepted: 27th February 2017

How to cite this article:
Chahar P.S. and Singh S.V. (2017): Stochastic Analysis of Two Unit Cold Standby System with Three Levels of Priority Unit. Annals of Natural Sciences, Vol. 3[1]: March, 2017: 38-47.

INTRODUCTION

Several authors including have analysed many engineering systems by using different sets of assumptions. Most of them considered two identical unit system in which an operative unit fails directly without passing to and other intermediate stages. But in the real practical situation there exists some systems in which an operative unit passes through different stages before its complete failure. For example, we consider a system of inverter with battery and a generator then we can observe that a battery of an inverter passes from different level of charging such as high, middle and low before completely down stage. We can also see that use of inverter is more economical than generator so the priority is to be given to inverter with respect to operation and repair.

Keeping the above view, we in this chapter analyse a two unit cold standby system in which first unit is priority and the second is treated as ordinary unit.

Using regenerative point technique with Markov renewal process, the following reliability characteristics of interest are obtained.

MODEL DESCRIPTION AND ASSUMPTIONS

1. The system comprises of two non-identical units in parallel configuration. Initially first unit is operative and the second is kept as cold standby. The first unit gets priority in operation as well as in repair while which the second unit is treated as ordinary.

2. The first unit starts operation from higher level and then passes through middle and lower level before reaching to failure stage. Similarly, if the charging process is going on then an operative unit reaches from lower to middle and from middle to high level. The priority unit can reach to failure stage only from middle level.
3. Probability that an operative unit will be in higher, middle and lower level of stages are fixed.
4. A single repair facility is available in the system to repair the failed priority and ordinary unit.
5. The failure time distributions of priority and ordinary units are exponential. Also, the rate of transition form higher to middle, middle to lower, lower to middle and middle to higher are exponential with different parameters.
6. The repair time distributions for priority and ordinary units are general.

NOTATION AND SYMBOLS

N_{POH} : Normal priority unit kept as operative and in higher level
N_{POM} : Normal priority unit kept as operative and in middle level
N_{POL} : Normal priority unit kept as operative and in lower level
N_{OO} : Normal ordinary unit kept as operative
N_{OCS} : Normal ordinary unit kept as cold standby
F_{pr} : Failed priority unit under repair
F_{or} : Failed ordinary unit under repair
F_{owr} : Failed ordinary unit waiting for repair
α_1 : Constant rate of transition of normal unit from higher tomiddle level
α_2 : Constant rate of transition of normal unit from middle to lower level
α_3 : Constant rate of transition of normal unit from lower level to failed stage
β : Constant failure rate of ordinary operative unit
δ_1 : Constant rate of transition of normal unit from middle to higher level
δ_2 : Constant rate of transition of normal unit from lower to middle level
δ_3 : Constant rate of transition of normal unit from lower to higher level
g(.), G(.) : pdf and cdf of the distribution of time to repair priority unit
h(.), H(.) : pdf and cdf of the distribution of time to repair ordinary unit
p_1 : Probability that repaired priority unit goes into higher level
p_2 : Probability that repaired priority unit goes into middle level
p_3 : Probability that repaired priority unit goes into lower level

Using the above notation and symbols the possible states of the system are

Up States:

$S_0 \equiv (N_{POH}, N_{OCS})$ $S_1 \equiv (N_{POM}, N_{OCS})$ $S_2 \equiv (N_{POL}, F_{OCS})$
$S_3 \equiv (F_{pr}, N_{OO})$ $S_5 \equiv (N_{POH}, F_{or})$ $S_6 \equiv (N_{POM}, F_{or})$

Down States:

$S_4 \equiv (F_{pr}, F_{owr})$

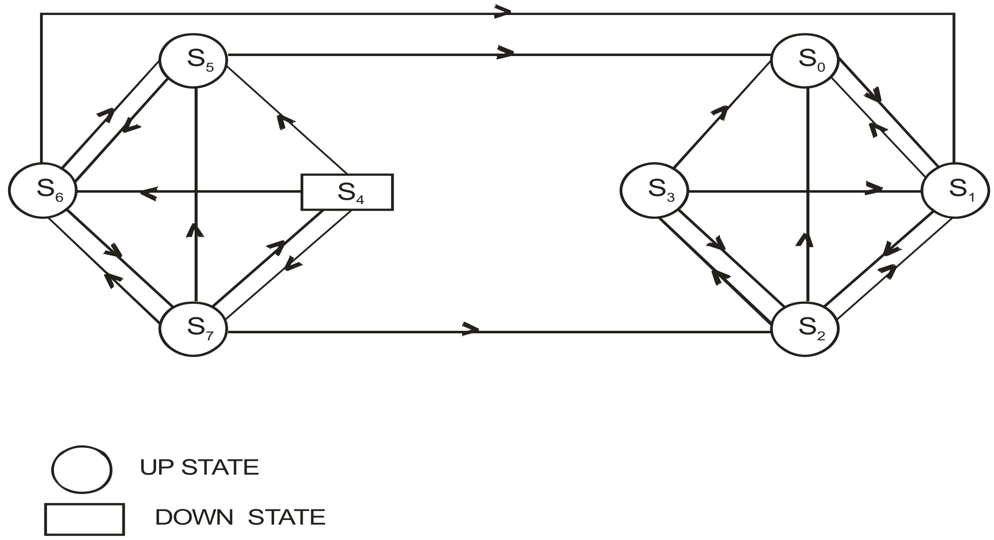

Fig. 1: The transitions between the various states

TRANSITION PROBABILITIES

Let T_0 (=0), T_1,T_2,.... be the epochs at which the system enters the states $S_i \in E$. Let X_n denotes the state entered at epoch T_{n+1} i.e. just after the transition of T_n. Then $\{T_n,X_n\}$ constitutes a Markov-renewal process with state space E and

$Q_{ik}(t) = Pr[X_{n+1} = S_k, T_{n+1} - T_n \leq t \mid X_n = S_i]$ (1)

is semi Markov-Kernal over E. The stochastic matrix of the embedded Markov chain is

$P = p_{ik} = \lim_{t\to\infty} Q_{ik}(t) = Q(\infty)$ (2)

The non-zero elements of $Q_{ik}(t)$ are given below:

$p_{ik} = \lim_{t\to\infty} Q_{ik}(t)$

$p_{01} = 1$

$p_{10} = \dfrac{\delta_1}{\alpha_2 + \delta_1}$

$p_{12} = \dfrac{\alpha_2}{\alpha_2 + \delta_1}$

$p_{20} = \dfrac{\delta_3}{\alpha_3 + \delta_2 + \delta_3}$

$p_{21} = \dfrac{\delta_2}{\alpha_3 + \delta_2 + \delta_3}$

$p_{23} = \dfrac{\alpha_3}{\alpha_3 + \delta_2 + \delta_3}$

$p_{30} = p_1. g^*(\beta)$

$p_{31} = p_2. g^*(\beta)$

$p_{32} = p_3. g^*(\beta)$

$p_{34} = 1 - g^*(\beta)$

$p_{45} = p_1$

$p_{46} = p_2$

$p_{47} = p_3$

$p_{50} = h^*(\alpha_1)$

$p_{56} = 1 - h^*(\alpha_1)$

$p_{61} = h^*(\alpha_2+\delta_1)$

$p_{65} = \frac{\delta_1}{\alpha_2 + \delta_1}[1 - h^*(\alpha_2+\delta_1)]$ $\quad p_{67} = \frac{\alpha_2}{\alpha_2 + \delta_1}[1 - h^*(\alpha_2+\delta_1)]$

$p_{72} = h^*(\alpha_3+\delta_2+\delta_3)$ $\quad p_{74} = \frac{\alpha_3}{\alpha_3 + \delta_2 + \delta_3}[1 - h^*(\alpha_3+\delta_2+\delta_3)]$

$p_{75} = \frac{\delta_3}{\alpha_3 + \delta_2 + \delta_3}[1 - h^*(\alpha_3+\delta_2+\delta_3)]$

$p_{76} = \frac{\delta_2}{\alpha_3 + \delta_2 + \delta_3}[1 - h^*(\alpha_3+\delta_2+\delta_3)]$

$p^{(4)}_{35} = p_1.[1 - \tilde{G}(\beta)]$ $\quad p^{(4)}_{36} = p_2.[1 - \tilde{G}(\beta)]$

$p^{(4)}_{37} = p_3.[1 - \tilde{G}(\beta)]$

$p^{(6)}_{51} = \frac{\alpha_1}{\alpha_1 - \alpha_2}[\tilde{H}(\alpha_2) - \tilde{H}(\alpha_1)]$

$p^{(7)}_{62} = \frac{\alpha_2}{\alpha_2 - \alpha_3}[\tilde{H}(\alpha_3) - \tilde{H}(\alpha_2)]$

$p^{(7)}_{64} = \frac{\alpha_2}{\alpha_2 - \alpha_3}[1 - \tilde{H}(\alpha_3)] - \frac{\alpha_3}{\alpha_2 - \alpha_3}[1 - \tilde{H}(\alpha_2)]$

$p^{(6,7)}_{52} = \frac{\alpha_1\alpha_2}{(\alpha_1 - \alpha_2)(\alpha_2 - \alpha_3)(\alpha_1 - \alpha_3)}[(\alpha_1-\alpha_2)\tilde{H}(\alpha_3)$

$+ (\alpha_2-\alpha_3)\tilde{H}(\alpha_1) - (\alpha_1-\alpha_3)\tilde{H}(\alpha_2)]$

$p^{(6,7)}_{54} = \frac{\alpha_1\alpha_2}{(\alpha_1 - \alpha_2)(\alpha_2 - \alpha_3)(\alpha_1 - \alpha_3)}[\{1 - \tilde{H}(\alpha_3)\}(\alpha_1-\alpha_2)]$

$+ \frac{\alpha_2\alpha_3}{(\alpha_1 - \alpha_2)(\alpha_2 - \alpha_3)(\alpha_1 - \alpha_3)}[\{1 - \tilde{H}(\alpha_1)\}(\alpha_2-\alpha_3)]$

$- \frac{\alpha_1\alpha_3}{(\alpha_1 - \alpha_2)(\alpha_2 - \alpha_3)(\alpha_1 - \alpha_3)}[\{1 - \tilde{H}(\alpha_2)\}(\alpha_1-\alpha_3)]$ (3-33)

From the above probabilities the following relation can be easily verifiesas;

$p_{01} = 1$ $\quad p_{10} + p_{12} = 1$

$p_{20} + p_{21} + p_{23} = 1$ $\quad p_{30} + p_{31} + p_{32} + p_{34} = 1$

$p_{45} + p_{46} + p_{47} = 1$

$p_{50} + p_{56} = 1 = p_{61} + p_{65} + p_{67}$

$p_{72} + p_{74} + p_{75} + p_{76} = 1 = p_{30} + p_{31} + p_{32} + p^{(4)}_{35} + p^{(4)}_{37}$

$p_{50} + p^{(6)}_{51} + p^{(6,7)}_{52} + p^{(6,7)}_{54} = 1 = p_{61} + p^{(7)}_{62} + p^{(7)}_{64} + p_{65}$ (34-41)

MEAN SOJOURN TIMES

The mean time taken by the system in a particular state S_i before transiting to any other state is known as mean sojourn time and is defined as-

$\mu_i = {}_0\int^{\infty} P[T>t]\, dt$ (42)

Using this we can obtain the following expression:

$\mu_0 = \dfrac{1}{\alpha_1}$ $\mu_1 = \dfrac{1}{\alpha_2+\delta_1}$ $\mu_2 = \dfrac{1}{\alpha_3+\delta_2+\delta_3}$

$\mu_3 = \dfrac{1}{\beta}[1 - g^*(\beta)]$ $\mu_4 = {}_0\int^{\infty} t.g(t)\, dt$

$\mu_5 = \dfrac{1}{\alpha_1}[1 - h^*(\alpha_1)]$ $\mu_6 = \dfrac{1}{\alpha_2+\delta_1}[1 - h^*(\alpha_2+\delta_1)]$

$\mu_7 = \dfrac{1}{\alpha_3+\delta_2+\delta_3}[1 - h^*(\alpha_3+\delta_2+\delta_3)]$ (43-50)

RELIABILITY OF THE SYSTEM

To obtain $R_i(t)$, the probability that starting from S_i the system will not fail upto time t, the failed state S_4 is taken to be absorbing state. Then by the probabilistic arguments following recursive relations can be obtained

$R_0(t) = Z_0(t) + q_{01}(t)©R_1(t)$

$R_1(t) = Z_1(t) + q_{10}(t)©R_0(t) + q_{12}(t)©R_2(t)$

$R_2(t) = Z_2(t) + q_{20}(t)©R_0(t) + q_{21}(t)©R_1(t) + q_{23}(t)©R_3(t)$

$R_3(t) = Z_3(t) + q_{30}(t)©R_0(t) + q_{31}(t)©R_1(t) + q_{32}(t)©R_2(t)$
(51-54)

Taking Laplace Stieltjes transform of relations and solving it for $R^*_0(s)$ we get

$R_0^*(s) = N_1(s)/ D_1(s)$ (55)

Where,

$N_1(s) = \{(1 - q^*_{23}q^*_{32}) - q^*_{12}(q^*_{23}q^*_{31} + q^*_{21})\}Z^*_0$

$+ q^*_{01}Z^*_1(1 - q^*_{23}q^*_{32}) + q^*_{01}q^*_{12}Z^*_2$ (56)

and

$D_1(s) = (1 - q^*_{23}q^*_{32}) - q^*_{12}(q^*_{23}q^*_{31} + q^*_{21}) - q^*_{01}q^*_{10}(1 - q^*_{23}q^*_{32})$

$- q^*_{01}q^*_{12}(q^*_{23}q^*_{30} + q^*_{20})$ (57)

Now,

$D_1(0) = 1 - p_{23}p_{32} - p_{23}p_{31}p_{12} - p_{21}p_{12} - p_{01}p_{10} - p_{01}p_{10}p_{23}p_{32}$

$p_{01}p_{12}p_{23}p_{30} - p_{01}p_{12}p_{20})$

$= (1 - p_{23}p_{32})(1 - p_{10}) - p_{12}(p_{20} + p_{21} + p_{23}p_{30} + p_{23}p_{31})$

$= (1 - p_{23}p_{32})p_{12} - p_{12}(p_{20} + p_{21} + p_{23}p_{30} + p_{23}p_{31})$

$= p_{12}[1 - p_{23}p_{32} - p_{20} - p_{21} - p_{23}p_{30} - p_{23}p_{31}]$

$= 0$ (58)

Now, the coefficient of m_{ij}'s in $D'_1(0)$ are

m_{ij}	Coefficients
m_{01}	$p_{10}(1 - p_{23}p_{32}) + p_{12}(p_{20} + p_{23}p_{30})$
m_{10}	$1 - p_{23}p_{32}$
m_{12}	$1 - p_{23}p_{32}$
m_{20}	p_{12}
m_{21}	p_{12}
m_{23}	p_{12}
m_{30}	$p_{12}p_{23}$
m_{31}	$p_{12}p_{23}$
m_{32}	$p_{12}p_{23}$

Then,

$$D'_1(0) = m_{01}\{ p_{10}(1 - p_{23}p_{32}) + p_{12}(p_{20} + p_{23}p_{30})\}$$

$$+ (m_{10} + m_{12})(1 - p_{23}p_{32}) + p_{12}(m_{20} + m_{21} + m_{23})$$

$$+ p_{12}p_{23}(m_{30} + m_{31} + m_{32})$$

$$= \mu_0\{ p_{10}(1 - p_{23}p_{32}) + p_{12}(p_{20} + p_{23}p_{30})\} + \mu_1(1 - p_{23}p_{32})$$

$$+ p_{12}\mu_2 + p_{12}p_{23}\mu_3 \quad (59)$$

Therefore, the reliability of the system is

$$R^*(0) = N_1/D_1 \quad (60)$$

Where

$$N_1 = N_1(0) = \{1 - p_{23}p_{32} - p_{23}p_{31}p_{12} - p_{21}p_{12}\}\mu_0 + \mu_1(1 - p_{23}p_{32})$$

$$+ p_{12}\mu_2 \quad (61)$$

and D_1 is same as in (59).

AVAILABILITY ANALYSIS

System availability is defined as

$A_i(t)$ = Pr[Starting from state S_i the system is available at epoch t without passing through any regenerative state]

and

$M_i(t)$ = Pr[Starting from up state S_i the system remains up till epoch t without passing through any regenerative up state]

Thus,

$M_0(t) = e^{-\alpha_1 t}$ $\qquad$ $M_1(t) = e^{-(\alpha_2+\delta_1)t}$

$M_2(t) = e^{-(\alpha_3+\delta_2+\delta_3)t}$ $\qquad$ $M_3(t) = e^{-\beta t}\,\overline{G}(t)$

$M_5(t) = e^{-\alpha_1 t}\,\overline{H}(t)$ $\qquad$ $M_6(t) = e^{-(\alpha_2+\delta_1)t}\,\overline{H}(t)$

$M_7(t) = e^{-(\alpha_3+\delta_2+\delta_3)t}\,\overline{H}(t)$ (62-68)

Now, obtaining $A_i(t)$ by using elementary probability argument;

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t)$

$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{12}(t)©A_2(t)$

$A_2(t) = M_2(t) + q_{20}(t)©A_0(t) + q_{21}(t)©A_1(t) + q_{23}(t)©A_3(t)$

$A_3(t) = M_3(t) + q_{30}(t)©A_0(t) + q_{31}(t)©A_1(t) + q_{32}(t)©A_2(t)$

$+ q^{(4)}{}_{35}(t)©A_5(t) + q^{(4)}{}_{36}(t)©A_6(t) + q^{(4)}{}_{37}(t)©A_7(t)$

$A_5(t) = M_5(t) + q_{50}(t)©A_0(t) + q^{(6)}{}_{51}(t)©A_1(t) + q^{(6,7)}{}_{52}(t)©A_2(t)$

$A_6(t) = M_6(t) + q_{65}(t)©A_5(t) + q^{(7)}{}_{62}(t)©A_2(t) + q_{61}(t)©A_1(t)$

$A_7(t) = M_7(t) + q_{72}(t)©A_2(t) + q_{75}(t)©A_5(t) + q_{76}(t)©A_6(t)$ (69-75)

Taking Laplace transform of above equation (69-75),and solving for pointwise availability $A^*_0(s)$, we get

$$A^*_0(s) = \frac{N_2(s)}{D_2(s)} \quad (76)$$

$N_2(s) = (M^*_0 + q^*_{01}M^*_1).A + q^*_{01}q^*_{12}(M^*_2 + B) - M^*_0q^*_{12}D$ (77)

and

$D_2(s) = (1 - q^*_{01}q^*_{10}).A - q^*_{12}.D - q^*_{12}.C$ (78)

By taking the limit $s\to 0$ in the relation (153), one gets the value of $D_2(0) = 0$, therefore the steady state availability of the system when it starts operations from S_0 is

$$A_0(\infty) = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s.\, A_0^*(s) = N_2(0)/D'_2(0) = N_2/D_2 \quad (79)$$

Where

$N_2 = N_2(0) = (\mu_0 + \mu_1).[1 - \{(p_{32} + p_{35}p_{52} + p_{36}p_{62} + p_{37}p_{72})$

$+ p_{52}(p_{36}p_{65} + p_{37}p_{75}) + p_{37}p_{76}(p_{65}p_{52} + p_{62})\}p_{23}]$

$+ p_{12}(\mu_2 + p_{23}\{(\mu_3 + p_{35}\mu_5 + p_{36}\mu_6 + p_{37}\mu_7)$

$+ \mu_5(p_{36}p_{65} + p_{37}p_{75}) + p_{37}p_{76}(\mu_6 + p_{65}\mu_5\})$

$- \mu_0p_{12}[\{(p_{31} + p_{35}p_{51} + p_{36}p_{61}) + (p_{36}p_{65} + p_{37}p_{75})p_{51}$

$+ p_{37}p_{76}(p_{65}p_{51} + p_{61}\}p_{23} + p_{21}]$ (80)

and using $m_i = \Sigma m_{ij}$, we have

$D_2 = [1 - \{(p_{32} + p_{35}p_{52} + p_{36}p_{62} + p_{37}p_{72})$

$+ p_{52}(p_{36}p_{65} + p_{37}p_{75}) + p_{37}p_{76}(p_{65}p_{52} + p_{62})\}p_{23}](p_{10}m_0 + m_1)$

$+ p_{12}m_2 + p_{12}p_{23}m_3 + p_{12}p_{23}\{(p_{36}p_{65} + p_{37}p_{75}) + p_{35}\}$

$+ p_{12}p_{23}[\{(p_{36} + p_{37}p_{76})m_6 + p_{12}p_{23}p_{37}m_7$ (81)

BUSY PERIOD ANANLYSIS

Let us define $W_i(t)$ as the probability that the system is under repair by repair facility in state $S_i \varepsilon E$ at time t without transiting to any regenerative state. Therefore

$W_1(t) = e^{-\delta_1 t}$ $W_2(t) = e^{-(\delta_2+\delta_3)t}$

$W_3(t) = \overline{G}(t)$ $W_5(t) = \overline{H}(t)$

$W_6(t) = e^{-\delta_1 t}\,\overline{H}(t)$ $W_7(t) = e^{-(\delta_2+\delta_3)t}\,\overline{H}(t)$ (82-87)

Also let $B_i(t)$ is the probability that the system is under repair by repair facility at time t, Thus the following recursive relations among $B_i(t)$'s can be obtained as ;

$B_0(t) = q_{01}(t)©B_1(t)$

$B_1(t) = W_1(t) + q_{10}(t)©B_0(t) + q_{12}(t)©B_2(t)$

$B_2(t) = W_2(t) + q_{20}(t)©B_0(t) + q_{21}(t)©B_1(t) + q_{23}(t)©B_3(t)$

$B_3(t) = W_3(t) + q_{30}(t)©B_0(t) + q_{31}(t)©B_1(t) + q_{32}(t)©B_2(t)$

$+ q^{(4)}_{35}(t)©B_5(t) + q^{(4)}_{36}(t)©B_6(t) + q^{(4)}_{37}(t)©B_7(t)$

$B_5(t) = W_5(t) + q_{50}(t)©B_0(t) + q^{(6)}_{51}(t)©B_1(t) + q^{(6,7)}_{52}(t)©B_2(t)$

$B_6(t) = W_6(t) + q_{65}(t)©B_5(t) + q^{(7)}_{62}(t)©B_2(t) + q_{61}(t)©B_1(t)$

$B_7(t) = W_7(t) + q_{72}(t)©B_2(t) + q_{75}(t)©B_5(t) + q_{76}(t)©B_6(t)$ (88-94)

Taking Laplace transform of above equation (88-94), and solving for $B^*_0(s)$, we get

$B^*_0(s) = N_3(s)/D_3(s)$ (95)

Where $D_3(s)$ is same as $D_2(s)$ in (153) and

$N_3(s) = q^*_{01}W^*_1[1 - \{(q^*_{32} + q^*_{35}q^*_{52} + q^*_{36}q^*_{62} + q^*_{37}q^*_{72})$

$+ q^*_{52}(q^*_{36}q^*_{65} + q^*_{37}q^*_{75}) + q^*_{37}q^*_{76}(q^*_{65}q^*_{52} + q^*_{62})\}q^*_{23}]$

$+ q^*_{01}q^*_{12}(W^*_2 + q^*_{23}\{(W^*_3 + q^*_{35}W^*_5 + q^*_{36}W^*_6 + q^*_{37}W^*_7)$

$+ W^*_5(q^*_{36}q^*_{65} + q^*_{37}q^*_{75}) + q^*_{37}q^*_{76}(W^*_6 + q^*_{65}W^*_5\})$

$- q^*_{12}[\{(q^*_{31} + q^*_{35}q^*_{51} + q^*_{36}q^*_{61}) + (q^*_{36}q^*_{65} + q^*_{37}q^*_{75})q^*_{51}$

$+ q^*_{37}q^*_{76}(q^*_{65}q^*_{51} + q^*_{61}\}q^*_{23} + q^*_{21}]$ (96)

In this steady state, the fraction of time for which the repair facility is busy in repair is given by

$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} s\,B^*(s) = N_3(0)/D'_3(0) = N_3/D_3$ (97)

where D_3 is same as D_2 in (156) and

$N_3 = w_1[1 - \{(p_{32} + p_{35}p_{52} + p_{36}p_{62} + p_{37}p_{72}) + p_{52}(p_{36}p_{65} + p_{37}p_{75})$

$+ p_{37}p_{76}(p_{65}p_{52} + p_{62})\}p_{23}] + p_{01}p_{12}(w_2 + p_{23}\{(w_3 + p_{35}w_5$

$+ p_{36}w_6 + p_{37}w_7) + w_5(p_{36}p_{65} + p_{37}p_{75}) + p_{37}p_{76}(w_6 + p_{65}w_5\})$

$- p_{12}[\{(p_{31} + p_{35}p_{51} + p_{36}p_{61}) + (p_{36}p_{65} + p_{37}p_{75})p_{51}$

$+ p_{37}p_{76}(p_{65}p_{51} + p_{61}\}p_{23} + p_{21}]$ (98)

EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

Let we define, $V_i(t)$ as the expected number of visits by the repairman in (0,t] given that the system initially started from regenerative state S_i at t=0. Then following recurrence relations among $V_i(t)$'s can be obtained as;

$V_0(t) = Q_{01}(t)\$[1 + V_1(t)]$

$V_1(t) = Q_{10}(t)\$V_0(t) + Q_{12}(t)\$V_2(t)$

$V_2(t) = Q_{20}(t)\$V_0(t) + Q_{21}(t)\$V_1(t) + Q_{23}(t)\$V_3(t)$

$V_3(t) = Q_{30}(t)\$V_0(t) + Q_{31}(t)\$V_1(t) + Q_{32}(t)\$V_2(t) + Q^{(4)}_{35}(t)\$V_2(t)$

$+ Q^{(4)}_{36}t)\$V_6t) + Q^{(4)}_{37}(t)\$V_7(t)$

$V_5(t) = Q_{50}(t)\$V_0(t) + Q^{(6)}_{51}t)\$V_1(t) + Q^{(6,7)}_{52}t)\$V_2(t)$

$V_6(t) = Q_{65}(t)\$V_5(t) + Q^{(7)}_{62}(t)\$V_7(t) + Q_{61}(t)\$V_1(t)$

$V_7(t) = Q_{72}(t)\$V_2(t) + Q_{75}(t)\$V_5(t) + Q_{76}(t)\$V_6(t)$ (99-405)

Taking Laplace stieltjes transform of the above equations (99-105) and solving for $\tilde{V}_0(s)$ we get

$\tilde{V}_0(s) = N_4(s)/D_4(s)$ (106)

Where

$N_4(s) = \tilde{Q}_{01}[1 - \{(\tilde{Q}_{32} + \tilde{Q}_{35}\tilde{Q}_{52} + \tilde{Q}_{36}\tilde{Q}_{62} + \tilde{Q}_{37}\tilde{Q}_{72})$

$+ \tilde{Q}_{52}(\tilde{Q}_{36}\tilde{Q}_{65} + \tilde{Q}_{37}\tilde{Q}_{75}) + \tilde{Q}_{37}\tilde{Q}_{76}(\tilde{Q}_{65}\tilde{Q}_{52} + \tilde{Q}_{62})\}\ \tilde{Q}_{23}]$

$- \tilde{Q}_{01}\tilde{Q}_{12}[\{(\tilde{Q}_{31} + \tilde{Q}_{35}\tilde{Q}_{51} + \tilde{Q}_{36}\tilde{Q}_{61})$

$+ (\tilde{Q}_{36}\tilde{Q}_{65} + \tilde{Q}_{37}\tilde{Q}_{75})\ \tilde{Q}_{51} + \tilde{Q}_{37}\tilde{Q}_{76}(\tilde{Q}_{65}\tilde{Q}_{51} + \tilde{Q}_{61}\}\tilde{Q}_{23}$

$+ \tilde{Q}_{21}]$ (107)

and $D_4(s) = (1 - \tilde{Q}_{01}\tilde{Q}_{10}).A - \tilde{Q}_{12}.D - \tilde{Q}_{12}.C$ (108)

In steady state the number of visit per unit of time when the system starts after entrance into state S_0 is ;

$$V_0 = \lim_{t \to \infty} [V_0(t)/t] = \lim_{s \to 0} s\, \tilde{V}_0(s) = N_4/D_4 \qquad (109)$$

where D_4 is same as D_2 in (81) and

$$N_4 = [1 - \{(p_{32} + p_{35}p_{52} + p_{36}p_{62} + p_{37}p_{72}) + p_{52}(p_{36}p_{65} + p_{37}p_{75})$$

$$+ p_{37}p_{76}(p_{65}p_{52} + p_{62})\}p_{23}] - p_{01}p_{12}[\{(p_{31} + p_{35}p_{51} + p_{36}p_{61})$$

$$+ (p_{36}p_{65} + p_{37}p_{75})\, p_{51} + p_{37}p_{76}(p_{65}p_{51} + p_{61}\}p_{23} + p_{21}] \qquad (110)$$

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