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**ORIGINAL ARTICLE**

# Unsteady MHD Free Convection and Mass Transfer Flow of Visco-Elastic (Walters Liqiud Model-B) Through Porous Medium with Heat Source/Sink

**Dharam Singh**
Mangalmay Institute of Engineering & Technology, Greater Noida
Email: singh.dharam57@gmail.com

**ABSTRACT**

*In this section we concerned with unsteady two-dimensional MHD free convection and mass transfer flow of a visco-elastic, incompressible, electrically conducting, fluid through a porous medium bounded by a vertical infinite surface with heat source/sink and constant suction velocity and constant heat flux in the presence of a uniform magnetic field is presented. The effects of various parameters with the time on the velocity, temperature, and concentration distribution and skin friction are discussed with the help of tables and graphs.*

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## INTRODUCTION

The effect of variable permeability on combined free and forced convection in porous media was studied by Chandrasekhara and Namboodiri (1986). Later on mixed convection in porous media adjacent to a vertical uniform heat flux surface was studied by Joshi and Gebhart (1985). Heat and Mass transfer in a porous medium was discussed by Bejan and Khair (1985). The above problem was studied in presence of buoyancy effect by Trevisan and Bejan (1985). Lai and Kulacki (1990) studied the effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium. The free convection effect on the flow of an ordinary viscous fluid past an infinite porous vertical plate with constant suction and constant heat flux was investigated by Sharma (1991). The study of two-dimensional flow through porous medium bounded by a vertical porous surface with constant suction velocity and constant heat flux in presence of free convection current was studied by Sharma (1992). Convection in a porous medium with inclined temprature gradient was investigated by Nield (1994). The problem of mixed convection along an isothermal vertical plate in porous medium injection and suction was studied by Hooper *et al*. (1994). Achary *et al*. (2000) have discussed magnetic effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Varshney and Kaushlendra Kumar (2002) have studied unsteady effect on MHD free convection and mass transfer flow through porous medium with constant suction and constant heat flux. Recently, Varshney and Singh (2004) have discussed unsteady effect on MHD free convection and Mass transfer flow of Rivlin-Ericksen fluid through porous medium with constant suction and constant heat flux.

In this section our aim is to investigate effects on unsteady two-dimensional MHD free convection and mass transfer flow of a visco-elastic (Walters liquid model-B), incompressible electrically conducting fluid through a porous medium bounded by a vertically infinite surface with heat source/sink.

## FORMULATION OF THE PROBLEM

We consider unsteady two dimensional motion of visco-elastic (Walters's liquid model-B), incompressible, electrically conducting, fluid through a porous medium occupying semi infinite region of space bounded by a vertical infinite surface with heat sourc/sink and constant suction velocity and constant heat flux under the action of a uniform magnetic field is applied. The effect of induced magnetic field is neglected. The Reynolds number is assumed to be small. Further magnetic field is not strong enough to cause Joule heating. Hence, the term due to electrical dissipation is neglected for energy equation. The *X*-axis is taken along the surface in the upward direction and *Y*-axis is taken normal to it. The fluid properties are assumed constant except for the influence of density in the body force term. As the bounding surface is infinite length, all the variables are function of *Y.* Hence, by the usual boundary layer approximation the basic equations for unsteady flow through porous medium are:

$$\frac{\partial V}{\partial Y} = 0 \qquad ...(1)$$

$$\frac{\partial U}{\partial t_0} + V\frac{\partial U}{\partial Y} = \upsilon\left(1 - E\frac{\partial}{\partial t_0}\right)\frac{\partial^2 U}{\partial Y^2} + g\beta(T - T_\infty)$$

$$+ g\beta^*(C - C_\infty) - \sigma\frac{B_0^2}{\rho}U - \frac{\upsilon}{K_0}U \qquad ...(2)$$

$$\frac{\partial T}{\partial t_0} + V\frac{\partial T}{\partial Y} = \frac{\lambda}{\rho C_p}\frac{\partial^2 T}{\partial Y^2} + S^*(T - T_\infty) \qquad ...(3)$$

$$\frac{\partial C}{\partial t_0} + V\frac{\partial C}{\partial Y} = D\frac{\partial^2 C}{\partial Y^2} \qquad ...(4)$$

Where $U$ and $V$ are the corresponding velocity components along and perpendicular to the surface, $\rho$ is the density, g is the acceleration due to gravity, $\beta$ is the coefficient of volume expansion, $\beta^*$ is the coefficient of concentration expansion, $\upsilon$ is the Kinematics viscosity, $T_\infty$ is the temperature of the fluid in the free stream, $C_\infty$ is the concentration at infinite, $\sigma$ is the electrical conductivity, $B_0$ is the magnetic induction, $K_0$ is porosity parameter, $\lambda$ is the thermal conductivity, $D$ is the concentration diffusivity, $C_p$ is the specific heat at constant pressure, $S$ is the heat source parameter etc.

## METHOD OF THE SOLUTION

The equation of continuity (1) gives

$$V = \text{constant} = -V_0 \qquad ...(5)$$

Where $V_0 > 0$ corresponds to steady suction velocity at the surface. In view of equation (5), equations (2), (3) and (4) can be written as-

$$\frac{\partial U}{\partial t_0} - V_0 \frac{\partial U}{\partial Y} = \upsilon\left(1 - E\frac{\partial}{\partial t_0}\right)\frac{\partial^2 U}{\partial Y^2} + g\beta(T - T_\infty)$$

$$+ g\beta^*(C - C_\infty) - \sigma\frac{B_0^2}{\rho}U - \frac{\upsilon}{K_0}U \quad ...(6)$$

$$\frac{\partial T}{\partial t_0} - V_0 \frac{\partial T}{\partial Y} = \frac{\lambda}{\rho C_p}\frac{\partial^2 T}{\partial Y^2} + S^*(T - T_\infty) \quad ...(7)$$

$$\frac{\partial C}{\partial t_0} - V_0 \frac{\partial C}{\partial Y} = D\frac{\partial^2 C}{\partial Y^2} \quad ...(8)$$

The boundary conditions of the problem are:

$$\left.\begin{array}{l} U = 0, \quad \dfrac{dT}{dY} = -\dfrac{q}{\lambda}, \quad \dfrac{dC}{dY} = -\dfrac{m}{D} \quad \text{at} \quad Y = 0, \quad t_0 = 0 \\ U \to 0, \quad T \to T_\infty, \quad C \to C_\infty, \quad \text{as} \quad Y \to \infty, \quad t_0 > 0 \end{array}\right\} \quad ...(9)$$

On introducing the following non-dimensional quantities into equations (6), (7) and (8)

$f(\eta) = \dfrac{U}{V_0}$ (Velocity),

$\eta = \dfrac{V_0 Y}{\upsilon}$ (Distance),

$\Pr = \dfrac{\mu C_p}{\lambda}$ (Prandtl number)

$Sc = \dfrac{\upsilon}{D}$ (Schmidt number)

$t = \dfrac{V_0^2 t_0}{\upsilon}$ (Time)

$\theta = \dfrac{(T - T_\infty)V_0\lambda}{q\upsilon}$

$\phi = \dfrac{(C - C_\infty)V_0 D}{m\upsilon}$

$K = \dfrac{V_0^2 K_0}{\upsilon^2}$ (Porosity parameter)

$S = \dfrac{V_0^2 S^*}{\upsilon}$ (Heat source/sink parameter)

$M = \dfrac{\sigma B_0^2 \upsilon}{\rho V_0^2}$ (Magnetic number)

$$E_0 = \frac{EV_0^2}{\upsilon}$$ (Visco-elastic parameter)

$$Gr = g\beta \frac{q\upsilon^2}{V_0^4 \lambda}$$ (Grashoff number for heat transfer)

$$Gm = g\beta^* \frac{m\upsilon^2}{V_0^4 D}$$ (Grashoff number for mass transfer)

Where '$q$' is the heat flux per unit area and '$m$' is the mass flux per unit area. Dropping star (*) we get-

$$-\frac{\partial f}{\partial t} + \frac{\partial^2 f}{\partial \eta^2} + \frac{\partial f}{\partial \eta} - E_0 \frac{\partial^3 f}{\partial t \partial \eta^2} - f\left(\frac{1}{K} + M\right) = -Gr\theta - Gm\phi \quad ...(10)$$

$$-\Pr\frac{\partial \theta}{\partial t} + \frac{\partial^2 \theta}{\partial \eta^2} + \Pr\frac{\partial \theta}{\partial \eta} + S\Pr\theta = 0 \quad ...(11)$$

$$-Sc\frac{\partial \varphi}{\partial t} + \frac{\partial^2 \varphi}{\partial \eta^2} + Sc\frac{\partial \varphi}{\partial \eta} = 0 \quad ...(12)$$

The corresponding boundary conditions becomes

$$\left.\begin{array}{llll} \eta = 0, & f = 0, & \theta' = -1, & \phi' = -1 \\ \eta \to \infty, & f \to 0, & \theta \to 0, & \phi \to 0 \end{array}\right\} \quad ...(13)$$

Following Mitra (1980), we assume the solution of

$$\left.\begin{array}{l} f(\eta,t) = f_0(\eta)\, e^{-nt} \\ \theta(\eta,t) = \theta_0(\eta)\, e^{-nt} \\ \phi(\eta,t) = \phi_0(\eta)\, e^{-nt} \end{array}\right\} \quad ...(14)$$

Substituting equation (14) into the equations (10), (11) and (12), we find

$$(1+nE_0) f_0''' + f_0'' - \left\{\left(M + \frac{1}{K}\right) - n\right\} f_0 = -Gr\theta_0 - Gm\phi_0 \quad ...(14)$$

$$(1+nE_0)\ \theta_0'' + \Pr\theta_0' + (n+S)\Pr\theta_0 = 0 \quad ...(15)$$

$$(1+nE_0)\ \phi_0 + Sc\phi_0' + nSc\phi_0 = 0 \quad ...(16)$$

With corresponding boundary conditions

$$\left.\begin{array}{lllll} f_0 = 0, & \theta_0' = -1, & \phi_0' = -1 & \text{at} & \eta = 0 \\ f_0 \to 0, & \theta_0 \to 0, & \phi_0 \to 0, & \text{as} & \eta \to 0 \end{array}\right\} \quad ...(17)$$

Solving equations (15) to (17) under boundary conditions (18), we get-

$$f_0 = (A_4 Gr + A_5 Gm) e^{-A_3\eta} - A_4 Gr e^{-A_1\eta} - A_5 Gm e^{-A_2\eta} \quad ...(18)$$

$$\theta_0 = \frac{1}{A_1} e^{-A_1\eta} \quad ...(19)$$

$$\phi_0 = \frac{1}{A_2} e^{-A_2\eta} \quad ...(20)$$

Where,

$$A_1 = \frac{\Pr + \sqrt{\Pr^2 - 4(n+S)(1+nE_0)\Pr}}{2(1+nE_0)}$$

$$A_2 = \frac{Sc + \sqrt{Sc^2 - 4n(1+nE_0)Sc}}{2(1+nE_0)}$$

$$A_3 = \frac{1 + \sqrt{1 + 4(1+nE_0)\left(M + \frac{1}{K} - n\right)}}{2(1+nE_0)}$$

$$A_4 = \frac{1}{A_1\left[(1+nE_0)A_1^2 - A_1 - \left(M + \frac{1}{K} - n\right)\right]}$$

$$A_5 = \frac{1}{A_2\left[(1+nE_0)A_2^2 - A_2 - \left(M + \frac{1}{K} - n\right)\right]}$$

Hence, the equations for $f$, $\theta$ and $\phi$ will be as follows:

$$f = \left[(A_4 Gr + A_5 Gm) e^{-A_3\eta} - A_4 Gr e^{-A_1\eta} - A_5 Gm e^{-A_2\eta}\right].e^{-nt} \quad ...(21)$$

$$\theta = \frac{1}{A_2} e^{-A_2\eta}.e^{-nt} \quad ...(22)$$

$$\phi = \frac{1}{A_3} e^{-A_3\eta}.e^{-nt} \quad ...(23)$$

**Skin Friction:**

The skin friction coefficient at the surface is given by

$$\tau = \left( \frac{\tau_{xy}}{\rho V_0^2} \right)_{\eta=0}$$

$$\tau = \left[ A_4 Gr \left( A_1 - A_3 \right) + A_5 Gm \left( A_2 - A_3 \right) \right] . e^{-nt} \qquad \ldots(24)$$

## RESULTS AND DISCUSSION

The fluid velocity profile of boundary layer flow are tabulated in table 1 to 2, and plotted graphs in fig. 1-2 for $Gm$=2, Schmidt number $Sc$ = 0.6, Prandtl number Pr = 0.71, $n$ = 0.1, and different values of Grashoff number $Gr$, permeability parameter $K$, magnetic field parameter $M$, visco-elastic parameter $E_0$, heat source/sink parameter S and $t$.

Variation in velocity $f$ for visco-elastic fluid in unsteady flow is shown in tables from 1 to 2 & fig. from 1 to 2 and these table & figure and graphs also illustrate the effects of $Gr$, M, K, S & $E_0$ and time from 0 to 2.

By perusing graphs from I & VII at $t$ = 0 of fig. 1 separately it is found that the velocity increases sharply till η = 1.2, after its velocity decreases continuously with increasing in η it is conclude the fluid velocity increases with increasing $Gr$, $K$ *and* $S$ but velocity decreases with increasing visco-elastic parameter $E_0$ and $M$.

Comparing graphs I to VII of fig. 2, with graphs I to VII of fig. 1, it is notice that each velocity graphs of fig. 1-2, is higher. This means that the fluid velocity increases with increasing time $t$ from 0 to 2.

The temperature profile of boundary layer flow is tabulated in table 3, and plotted graphs in fig. 3 for $Gr$ = 5, $Gm$ = 2, permeability parameter ($K$) = 1.0, magnetic field parameter ($M$) = 1.0, $E_0$ = 0.1, $n$ = 0.1, Schmidt number $Sc$ = 0.6, time $t$ and different values of Prandtl number (Pr) and heat source/sink parameter (S). From this figure it is observed that the temperature decreases continuously with increasing in η and it is conclude the temperature decreases with increasing Prandtl number (Pr), but the temperature increases with increasing heat source/sink parameter (S).

The concentration profile of boundary layer flow is tabulated in table 4, and plotted graphs in fig. 4 for $Gr$ = 5, $Gm$ = 2, permeability parameter $K$ = 1.0, magnetic field parameter $M$ = 1.0, visco-elastic parameter $E_0$ = 0.1, $n$ = 0.1, Prandtl number Pr = 0.71, different values of Schmidt number $Sc$ and time $t$. From this figure it is observed that the concentration decreases continuously with increasing in η and it is conclude the concentration decreases with increasing Schmidt number $Sc$, but the concentration increases with increasing time $t$ from 0 to 2.

The skin friction profile is tabulated in table 5 and plotted graphs in fig. 5 for $Gm$ = 2, Prandtl number (Pr) = 0.71, $Sc$ = 0.6, $n$ = 0.1, S = 0.07 and different values of Grashoff number ($Gr$), permeability parameter ($K$), magnetic field parameter ($M$), and Visco-elastic parameter ($E_0$).

By perusing graphs from I & V of fig. 5 separately it is observed that the skin friction decreases gradually with increasing time $t$. Comparing graphs it is obvious that the skin friction increases with increasing $Gr$), permeability parameter ($K$), but skin friction decreases with increasing magnetic field parameter ($M$) and visco-elastic parameter ($E_0$).

## CONCLUSION

The theoretical solution for unsteady two-dimensional free convection and mass transfer flow of a visco-elastic, incompressible, electrically conducting, fluid through a porous medium bounded by a vertical infinite surface with heat source/sink and constant suction velocity and constant heat flux in the presence of uniform magnetic field. The solutions are in terms of exponential functions. The study concludes the following results.

1. The fluid velocity profile increases with increasing *Gr, K and S* but velocity decreases with increasing visco-elastic parameter $E_0$, and M.
2. The fluid velocity increases with increasing time $t$ from 0 to. 2, when the velocity decreases with increasing time $t$ from 0 to 2.
3. The temperature decreases with increases Prandtl number Pr, but the temperature increases with increasing heat source/sink.
4. The concentration decreases with increasing Schmidt number *Sc,* but the concentration increases with increasing time $t$ from 0 to 2.
5. The skin friction increases with increasing *Gr,* permeability parameter $K$ but skin friction decreases with increasing magnetic field parameter $M$ and visco-elastic parameter $E_0$.

**Table 1:** Values of velocity profile at $Sc$ = 0.6, $Gm$ = 2, Pr = 0.71, $n$ = 0.1, time $t$ = 0 and different values of $Gr$, $K$, $M$ visco-elastic parameter $E_0$ and S

| η | Graph-1 | Graph-2 | Graph-3 | Graph-4 | Graph-5 | Graph-6 | Graph-7 |
|---|---|---|---|---|---|---|---|
| 0 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 1 | 3.72768 | 5.39596 | 2.73182 | 4.58914 | 3.36328 | 2.58406 | 1.63083 |
| 2 | 2.93116 | 4.25518 | 2.04556 | 3.73775 | 2.86346 | 1.86432 | 1.08917 |
| 3 | 1.96470 | 2.86129 | 1.34848 | 2.54235 | 1.90007 | 1.13543 | 0.62700 |
| 4 | 1.27772 | 1.86690 | 0.87276 | 1.66218 | 1.30131 | 0.66923 | 0.35905 |
| 5 | 0.82584 | 1.21056 | 0.56339 | 1.07621 | 0.85330 | 0.39242 | 0.20971 |
| 6 | 0.53320 | 0.78406 | 0.36363 | 0.69522 | 0.55195 | 0.23041 | 0.12494 |
| 7 | 0.34427 | 0.50780 | 0.23477 | 0.44895 | 0.35659 | 0.13567 | 0.07557 |
| 8 | 0.22234 | 0.32894 | 0.15162 | 0.28996 | 0.23034 | 0.08015 | 0.04619 |

**Table 2:** Values of velocity profile at $Sc$ = 0.6, $Gm$ = 2, Pr = 0.71, $n$ = 0.1, time $t$ = 2 and different values of $Gr$, $K$, $M$ visco-elastic parameter $E_0$ and S

| η | Graph-1 | Graph-2 | Graph-3 | Graph-4 | Graph-5 | Graph-6 | Graph-7 |
|---|---|---|---|---|---|---|---|
| 0 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| 1 | 4.55300 | 6.59064 | 3.33665 | 5.60518 | 4.00762 | 3.15618 | 1.99191 |
| 2 | 3.58012 | 5.19728 | 2.49845 | 4.56530 | 3.47084 | 2.27708 | 1.33032 |
| 3 | 2.39969 | 3.49479 | 1.64703 | 3.10524 | 2.43433 | 1.38682 | 0.76582 |
| 4 | 1.56061 | 2.28024 | 1.06599 | 2.03020 | 1.61348 | 0.81740 | 0.43854 |
| 5 | 1.00868 | 1.47858 | 0.68812 | 1.31448 | 1.05054 | 0.47930 | 0.25614 |
| 6 | 0.65125 | 0.95766 | 0.44414 | 0.84915 | 0.68010 | 0.28142 | 0.15261 |
| 7 | 0.42049 | 0.62023 | 0.28674 | 0.54835 | 0.43952 | 0.16571 | 0.09230 |
| 8 | 0.27157 | 0.40177 | 0.18519 | 0.35416 | 0.28394 | 0.09789 | 0.05642 |

**Table 3:** Values of temperature profile at Gr = 5, $K$ = 1, $Sc$ = 0.6, $Gm$ = 2, $M$ = 1.0, $n$ = 0.1, time $t$ = 2 and different values of Pr and S.

| η | Graph-1 | Graph-2 | Graph-3 | Graph-4 |
|---|---|---|---|---|
| 0 | 2.8539275 | 1.8162502 | 2.0716697 | 1.3031653 |
| 1 | 1.8602735 | 0.9270833 | 1.1488723 | 0.5104501 |
| 2 | 1.2125807 | 0.4732186 | 0.6371225 | 0.1999434 |
| 3 | 0.7903956 | 0.2415488 | 0.3533249 | 0.0783179 |
| 4 | 0.5152030 | 0.1232957 | 0.1959410 | 0.0306771 |
| 5 | 0.3358244 | 0.0629348 | 0.1086617 | 0.0120162 |
| 6 | 0.2189002 | 0.0321243 | 0.0602598 | 0.0047068 |
| 7 | 0.1426855 | 0.0163975 | 0.0334179 | 0.0018436 |
| 8 | 0.0930066 | 0.0083699 | 0.0185323 | 0.0007222 |

**Table 4:** Values of concentration profile at $K$ = 1, Pr = 0.71, Gr = 5, $Gm$ = 2, $M$ = 1.0, $n$ = 0.1, $E_0$ = 0.1, S = 0.71 and different values of Schmidt number $Sc$ and time $t$

| η | Graph-1 | Graph-2 | Graph-3 | Graph-4 |
|---|---|---|---|---|
| 0 | 2.11324865 | 1.46446609 | 1.12701665 | 2.58112773 |
| 1 | 1.31655876 | 0.73981733 | 0.46406794 | 1.60804850 |
| 2 | 0.82021912 | 0.37374008 | 0.19108773 | 1.00181790 |
| 3 | 0.51099839 | 0.18880559 | 0.07868356 | 0.62413485 |
| 4 | 0.31835317 | 0.09538060 | 0.03239927 | 0.38883744 |
| 5 | 0.19833476 | 0.04818426 | 0.01334094 | 0.24224662 |
| 6 | 0.12356301 | 0.02434167 | 0.00549336 | 0.15092020 |
| 7 | 0.07698004 | 0.01229690 | 0.00226198 | 0.09402363 |
| 8 | 0.04795874 | 0.00621213 | 0.00093141 | 0.05857694 |

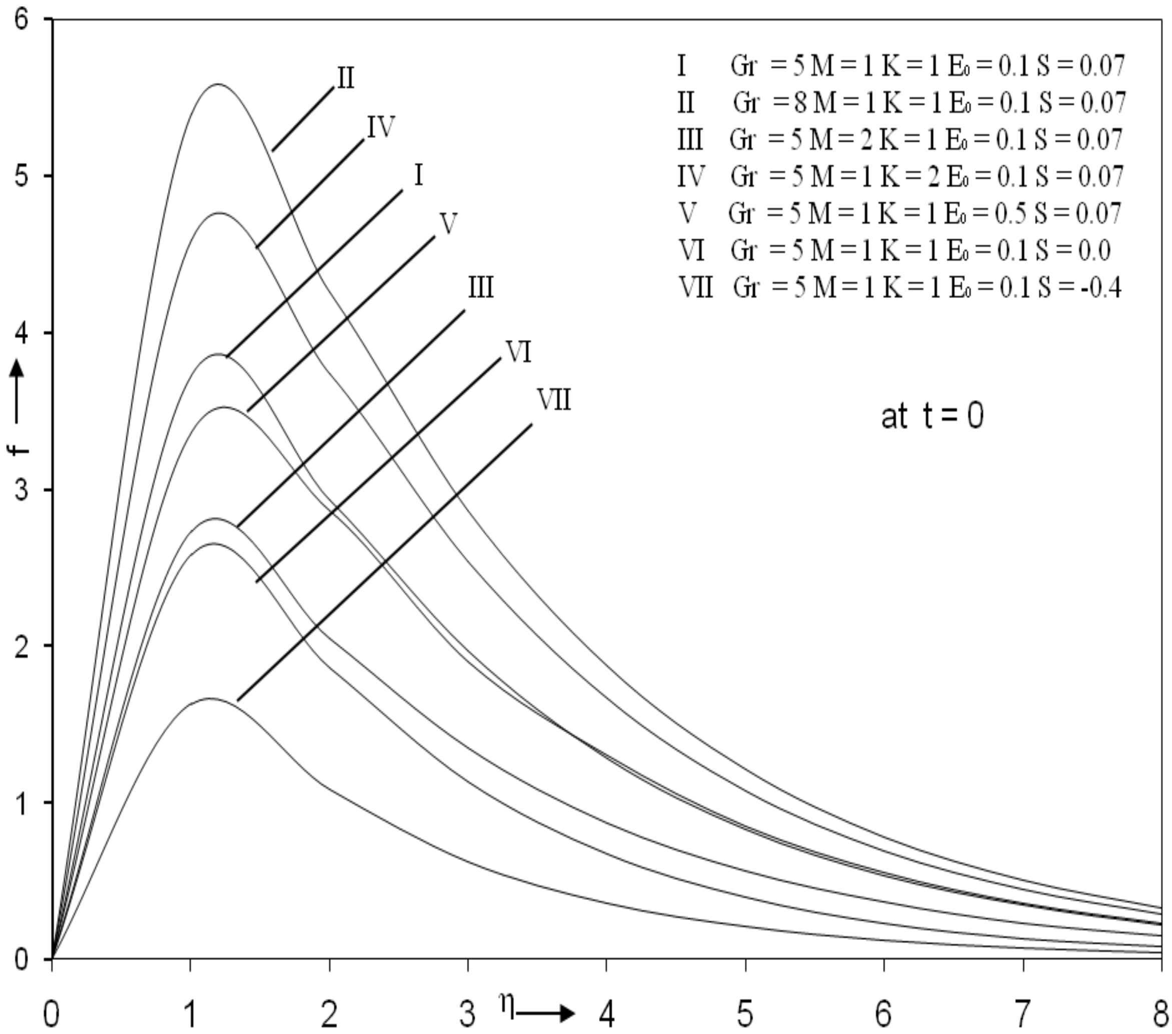


**Fig. 1:** The velocity profie for different value of Gr, M, K, S and $E_0$ at t=0

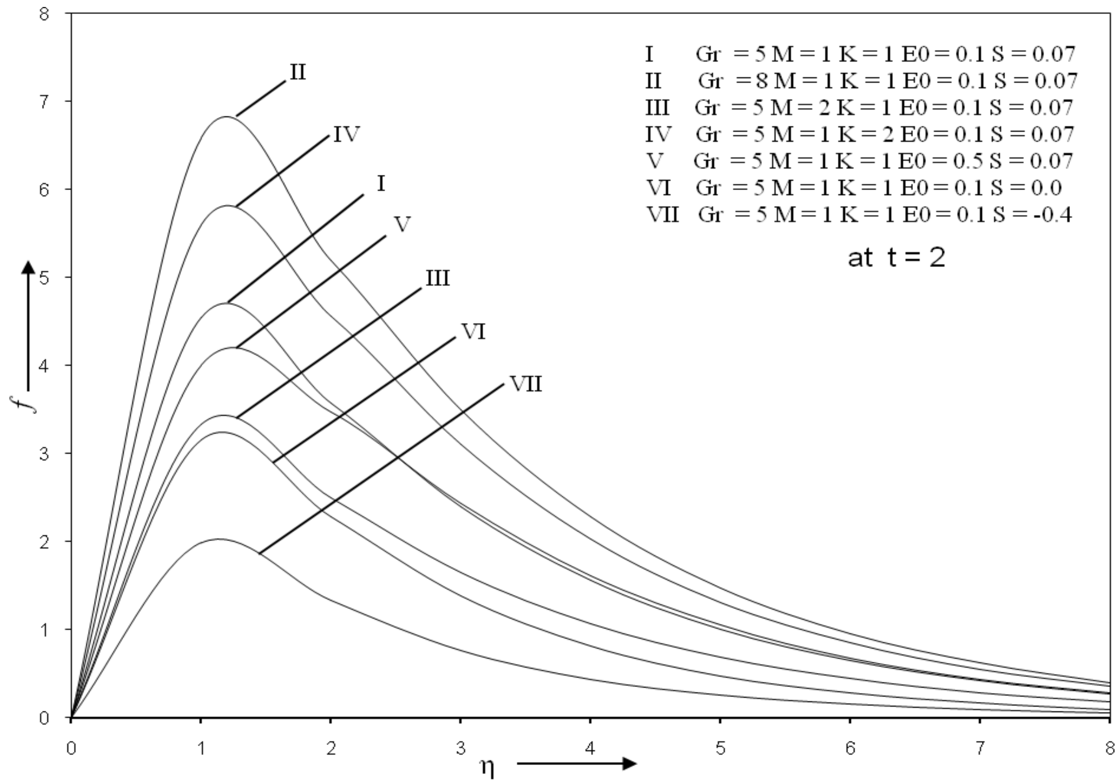


**Fig. 2:** The velocity profie for different value of Gr, M, K, S and $E_0$ at t=2

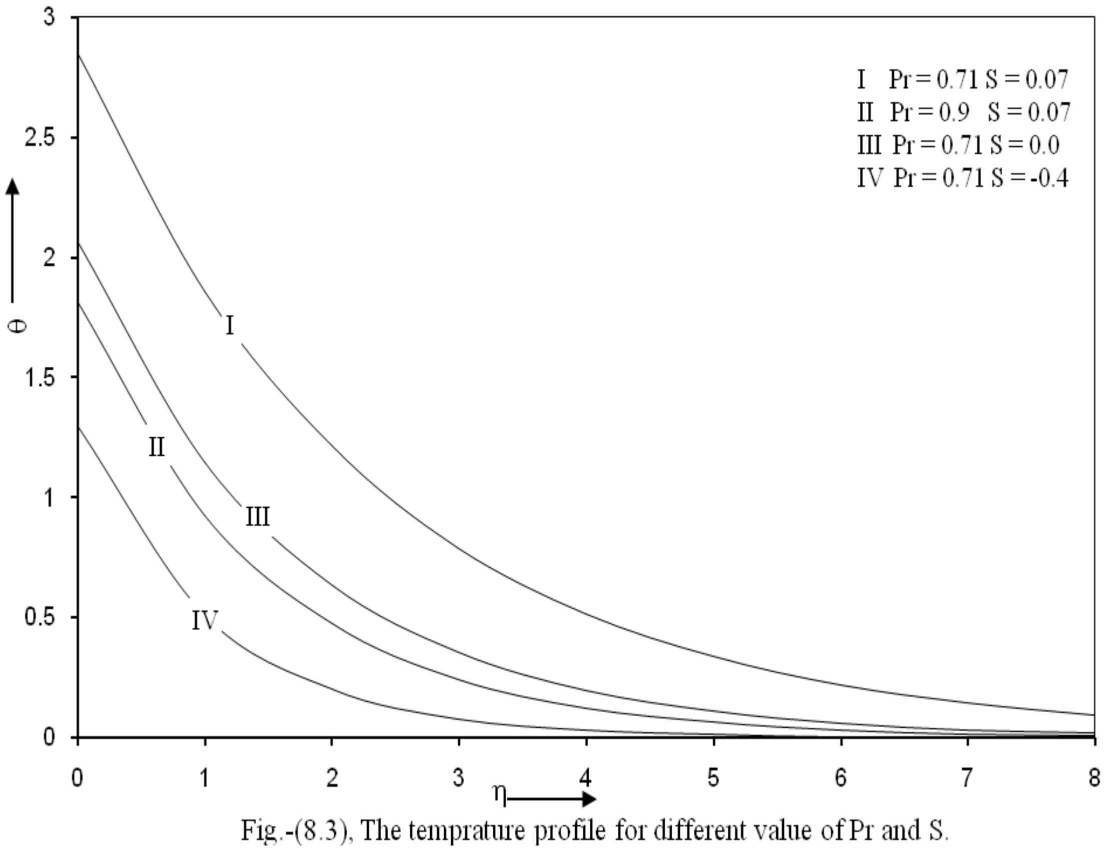


**Fig. 3:** The temperature profie for different value of Pr and S

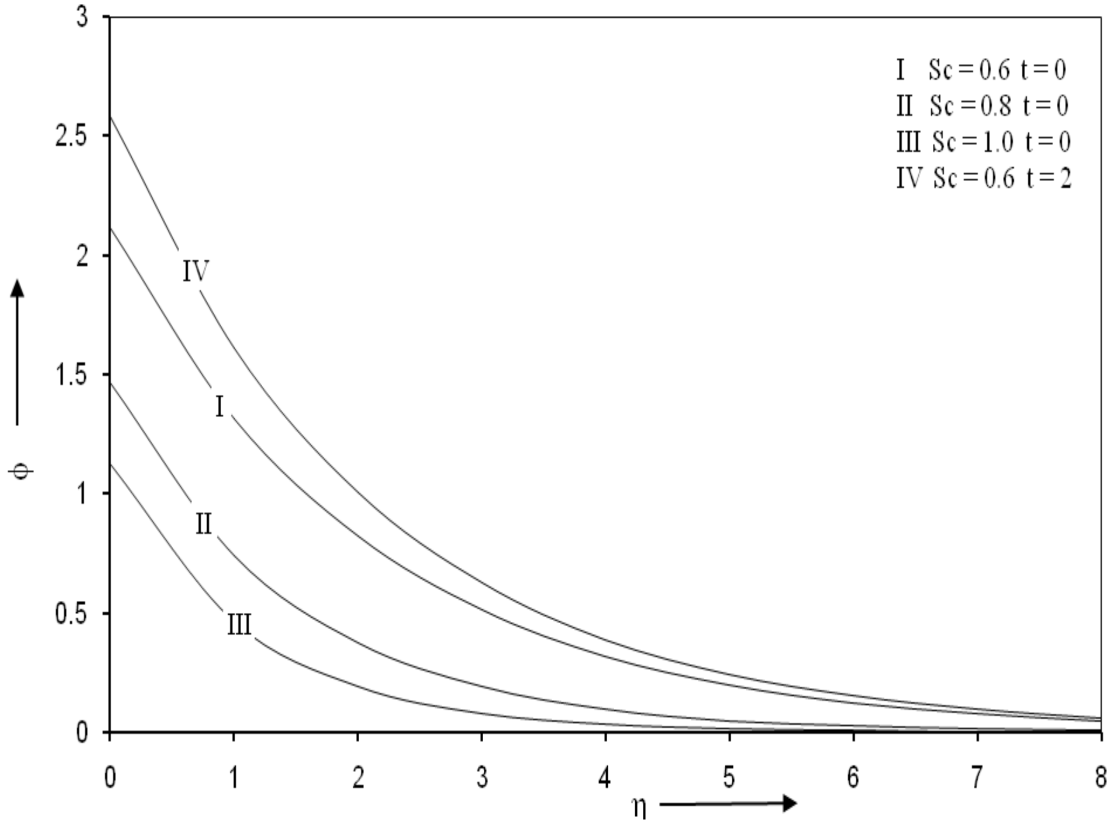


**Fig. 4:** The concentration profie for different value of t and Sc

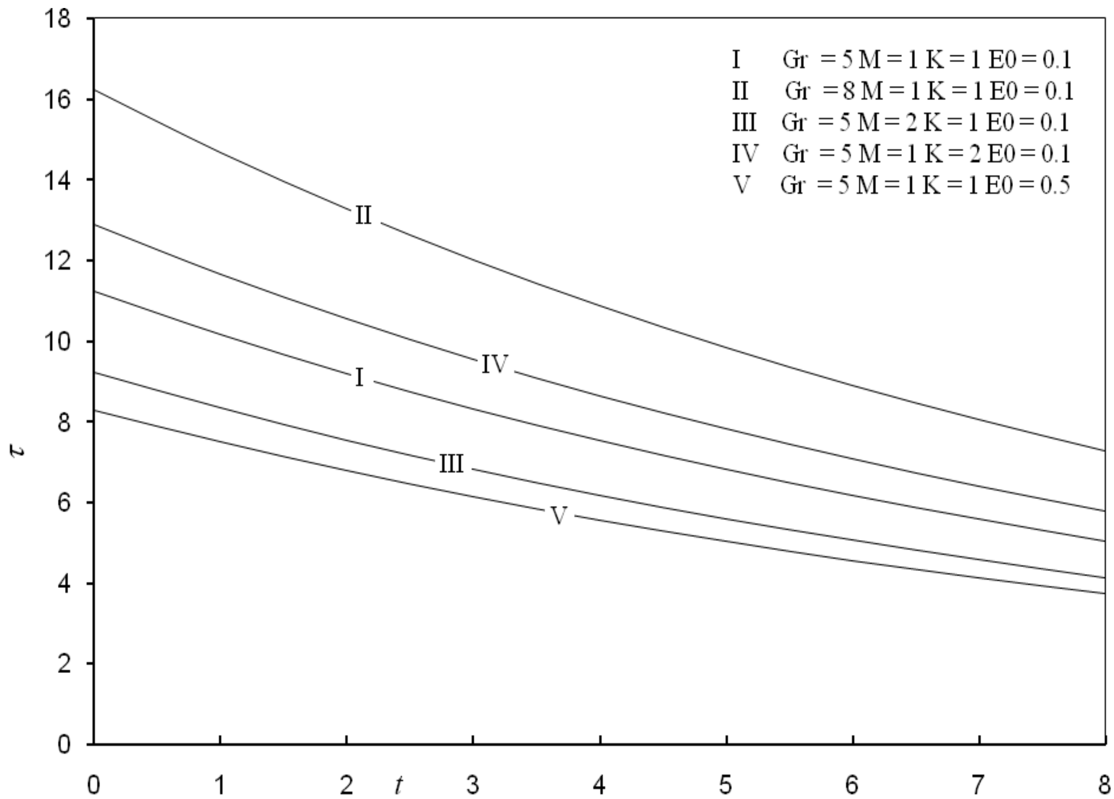


**Fig. 5:** The skin profie for different value of Gr, M, K and $E_0$

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