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ORIGINAL ARTICLE

Stochastic Analysis of a Two Unit Warm Standby System with Preventive Maintenance

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ABSTRACT

This chapter investigates a system consists of two identical units in which one is operative and the other is kept as warm standby. Whenever first unit fails the second unit becomes operative instantaneously. If both the units of the system are alive i.e. first unit is in operating position and the second is as standby, and if the operative unit works continuously up to a random amount of time then send this unit to preventive maintenance and in such a situation the standby unit at once comes into operation. This process of preventive maintenance is done by operator himself in which the unit will be in completely rest and some oiling and complete checkup of the unit is performed. There is no repair facility in the system but at the time of need an ordinary repairman will be called to attend the failed unit. If the ordinary repairman enters in the system within fixed amount of time known as patience time then it is O.K. otherwise an expert repairman will be called. Once any of the repairman enters in the system he will complete all the jobs related to the system. After the repair a unit works as good as new. The distribution of time to complete preventive maintenance follows exponential while the distributions of failure time, repair times and the time after continuous working, an operative unit is sent for preventive maintenance are arbitrary.

Key words: *Stochastic Analysis, Unit Warm Standby System, Preventive Maintenance*

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INTRODUCTION

Many authors including working in the field of reliability have analysed various engineering systems by assuming different sets of assumptions. Most of them analysed two unit standby systems with the assumption that a single repair facility is available in the system always and whenever an operative unit fails it comes into repair immediately.

But there exists many practical situations in which it is quite reasonable that the repair facility should not be available in the permanent capacity. The repairman should be call at the time of need on job basis.

Keeping the above view, we in this chapter analysed a two unit warm standby system with two types of repair facilities known as ordinary and expert repairman which are called at the time of need.

Using regenerative point technique with Markov renewal process, the following reliability characteristics of interest are obtained.

MODEL DESCRIPTION AND ASSUMPTIONS

The system consists of two identical units in which one is operative and the other is kept as warm standby.

1. Whenever first unit fails the second unit becomes operative instantaneously.

2. If both the units of the system are alive i.e. first unit is in operating position and the second is as standby, and if the operative unit works continuously up to a random amount of time then send this unit to preventive maintenance and in such a situation the standby unit at once comes into operation. This process of preventive maintenance is done by operator himself in which the unit will be in completely rest and some oiling and complete checkup of the unit is performed.
3. There is no repair facility in the system but at the time of need an ordinary repairman will be called to attend the failed unit. If the ordinary repairman enters in the system within fixed amount of time known as patience time then it is O.K. otherwise an expert repairman will be called. Once any of the repairman enters in the system he will complete all the jobs related to the system.
4. After the repair a unit works as good as new.
5. The distribution of time to complete preventive maintenance follows exponential while the distributions of failure time, repair times and the time after continuous working, an operative unit is sent for preventive maintenance are arbitrary.

NOTATION AND SYMBOLS

N_O	:	Normal unit kept as operative
N_S	:	Normal unit kept as warm standby
N_{pm}	:	Normal unit under preventive maintenance
N_{OC}	:	Operation of Normal unit is continued from earlier state
F_{ro}	:	Failed unit under repair by ordinaryrepairman
F_{re}	:	Failed unit under repair by expert repairman
F_{RO}	:	Repair of failed unit by ordinary repairman is continued from earlier state
F_{RE}	:	Repair of failed unit by expert repairman is continued from earlier state
F_{wro}	:	Failed unit waiting for repair by ordinary repairman
F_{wre}	:	Failed unit waiting for repair by expert repairman
β	:	Constant rate of completing preventive maintenance
$f_1(.)$, $F_1(.)$	:	pdf and cdf of failure time distribution of operative unit
$f_2(.)$, $F_2(.)$	:	pdf and cdf of failure time distribution of warm standby unit
$g_1(.)$, $G_1(.)$	:	pdf and cdf of time to complete repair of a failed unit by ordinary repairman
$g_2(.)$, $G_2(.)$	:	pdf and cdf of time to complete repair of a failed unit by expert repairman
$h(.)$, $H(.)$	:	pdf and cdf of patience time for ordinary repairman
$w(.)$, $W(.)$	:	pdf and cdf of availability time of ordinary repairman
$k(.)$, $K(.)$	:	pdf and cdf of time after completing which an operative unit is sent

for preventive maintenance using the above notation and symbols the possible states of the system are

Up States:

$S_0 \equiv (N_O, N_S)$ $S_1 \equiv (N_O, F_{wro})$ $S_2 \equiv (N_O, N_{pm})$
$S_3 \equiv (N_{OC}, F_{ro})$ $S_4 \equiv (N_{OC}, F_{re})$ $S_8 \equiv (N_O, F_{re})$
$S_{10} \equiv (N_O, F_{re})$ $S_{11} \equiv (N_O, F_{ro})$ $S_{12} \equiv (N_{OC}, N_S)$

Down States:

$S_5 \equiv (F_{re}, F_{wre})$ $S_6 \equiv (F_{RO}, F_{wro})$ $S_7 \equiv (F_{RE}, F_{wre})$
$S_9 \equiv (N_{pm}, F_{re})$

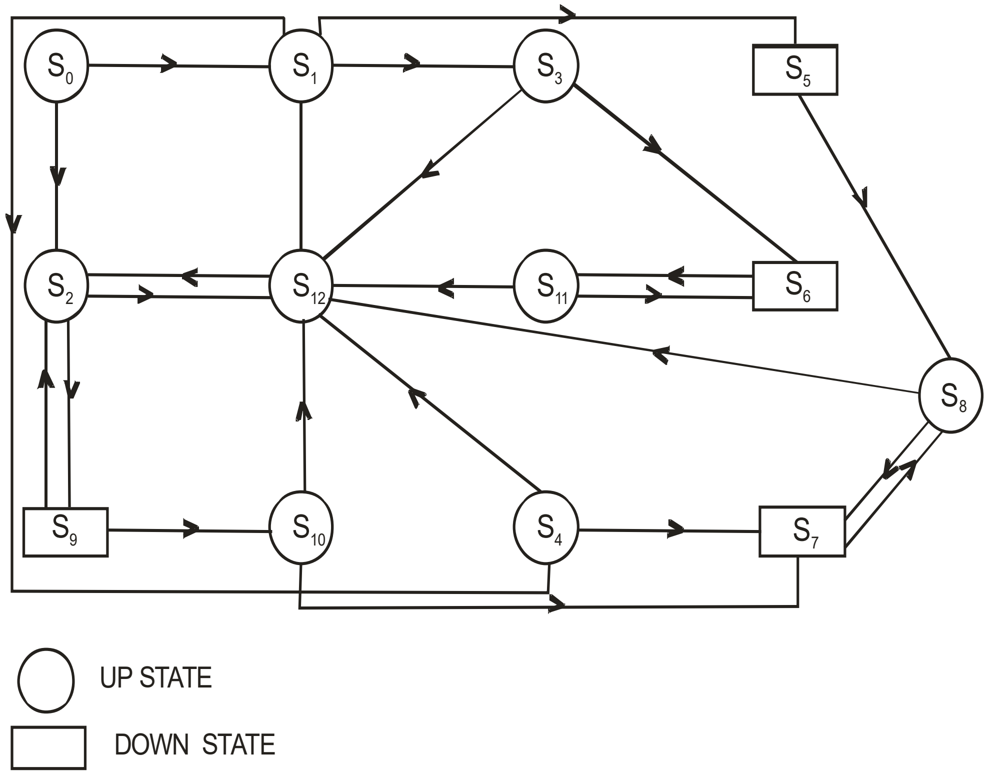

Fig. 1: The transitions between the various states

TRANSITION PROBABILITIES

Let T_0 (=0), $T_1, T_2, \ldots$ be the epochs at which the system enters the states $S_i \in E$. Let X_n denotes the state entered at epoch T_{n+1} i.e. just after the transition of T_n. Then $\{T_n, X_n\}$ constitutes a Markov-renewal process with state space E and

$$Q_{ik}(t) = Pr[X_{n+1} = S_k, T_{n+1} - T_n \leq t \mid X_n = S_i] \quad (1)$$

is semi Markov-Kernal over E. The stochastic matrix of the embedded Markov chain is

$$P = p_{ik} = Q(\infty) \quad (2)$$

The non-zero elements of $Q_{ik}(t)$ are given below ;

$p_{01} = {}_0\int^{\infty} \overline{K}(t)\overline{F}_1(t)\, f_2(t)\, dt + {}_0\int^{\infty} \overline{K}(t)\overline{F}_2(t)\, f_1(t)\, dt$

$p_{02} = {}_0\int^{\infty} \overline{F}_1(t)\, \overline{F}_2(t)\, k(t)\, dt$

$p_{15} = {}_0\int^{\infty} \overline{W}(t)\, \overline{H}(t)\, f_1(t)\, dt$

$p_{29} = f^*_1(\beta)$

$p_{58} = 1$

$p_{92} = g^*_2(\beta)$

$p^{(4)}_{17} = {}_0\int^{\infty} {}_0\int^{t} \overline{W}(w)\, dH(w)\, \overline{G}_2(t-w)\, dF_1(t)$

$p^{(3)}_{16} = {}_0\int^{\infty} {}_0\int^{t} \overline{H}(w)\, dW(w)\, \overline{G}_1(t-w)\, dF_1(t)$

$p^{(12)}_{21} = \beta\, {}_0\int^{\infty} {}_0\int^{t} e^{-\beta w}\, dw\, \overline{K}(t-w)\, dF_1(t)$

$p^{(12)}_{22} = \beta\, {}_0\int^{\infty} {}_0\int^{t} e^{-\beta w}\, dw\, \overline{F}_1(t)\, dK(t-w)$

$p^{(12)}_{18} = {}_0\int^{\infty} {}_0\int^{t} dG_2(w)\, \overline{K}(t-w)\, dF_1(t)$

$p^{(12)}_{82} = {}_0\int^{\infty} {}_0\int^{t} dG_2(w)\, \overline{F}_1(t)\, dK(t-w)$

$p^{(7)}_{88} = {}_0\int^{\infty} f_1(t)\, dG_2(t)$

$p^{(12)}_{11,1} = {}_0\int^{\infty}{}_0\int^{t} dG_1(w)\ \overline{K}(t-w)\ dF_1(t)$
$p^{(12)}_{11,2} = {}_0\int^{\infty}{}_0\int^{t} dG_1(w)\ \overline{F}_1(t)\ dK(t-w)$
$p^{(6)}_{11,11} = {}_0\int^{\infty} f_1(t)\ dG_1(t)$
$p^{(4,12)}_{12} = {}_0\int^{\infty}{}_0\int^{w}{}_0\int^{t} dH(u)\ \overline{W}(u)\ dG_2(w-u)\ \overline{F}_1(t)\ dK(t-w)$
$p^{(3,12)}_{12} = {}_0\int^{\infty}{}_0\int^{t}{}_0\int^{w} \overline{H}(t)\ dW(t)\ dG_1(w-t)\ \overline{F}_1\ dK(t-w)$
$p^{(4,12)}_{11} = {}_0\int^{\infty}{}_0\int^{t}{}_0\int^{w} \overline{W}(u)\ dH(u)\ dG_2(w-u)\ \overline{K}(t-w)\ dF_1(t)$
$p^{(3,12)}_{11} = {}_0\int^{\infty}{}_0\int^{t}{}_0\int^{w} \overline{H}(u)\ dW(u)\ dG_1(w-u)\ \overline{K}(t-w)\ dF_1(t)$
$p^{(4,7)}_{18} = {}_0\int^{\infty}{}_0\int^{t}{}_0\int^{w} \overline{W}(u)\ dH(u)\ dF_1(w)\ dG_2(t-u)$
$p^{(3,6)}_{1,11} = {}_0\int^{\infty}{}_0\int^{t}{}_0\int^{w} \overline{H}(u)\ dW(u)\ dF_1(w)\ dG_1(t-u)$
$p^{(10,12)}_{92} = {}_0\int^{\infty}{}_0\int^{t}{}_0\int^{w} \beta e^{-\beta u}\ du\ dG_2(w)\ \overline{F}_1(t-u)\ dK(t-w)$
$p^{(10,7)}_{98} = {}_0\int^{\infty}{}_0\int^{t}{}_0\int^{w} \beta e^{-\beta u}\ du\ dF_1(w-u) dG_2(u)$ (3-26)

MEAN SOJOURN TIME

The mean time taken by the system in a particular state S_i before transiting to any other state is known as mean sojourn time and is defined as

$$\mu_i = {}_0\int^{\infty} P[T>t]\ dt \quad (27)$$

Using this we can obtain the following expression:

$\mu_0 = {}_0\int^{\infty} \overline{F}_1(t)\ \overline{F}_2(t)\ \overline{K}(t)\ dt$ $\quad\mu_1 = {}_0\int^{\infty} \overline{W}(t)\ \overline{F}_1(t)\ \overline{H}(t)\ dt$

$\mu_2 = {}_0\int^{\infty} e^{-\beta t}\ \overline{F}_1(t)\ dt$ $\quad\mu_5 = {}_0\int^{\infty} \overline{G}_2(t)\ dt$

$\mu_8 = {}_0\int^{\infty} \overline{F}_1(t)\ \overline{G}_2(t)\ dt$ $\quad\mu_9 = {}_0\int^{\infty} e^{-\beta t}\ \overline{G}_2(t)\ dt$

$\mu_{11} = {}_0\int^{\infty} \overline{F}_1(t)\ \overline{G}_1(t)\ dt$ (28-34)

MEAN TIME TO SYSTEM FAILURE (MTSF)

To obtain the distribution function $\pi_i(t)$ of the time to system failure with starting state S_0.

$\pi_0(t) = Q_{01}(t)\$\pi_1(t) + Q_{02}(t)\$\pi_2(t)$

$\pi_1(t) = Q_{15}(t) + Q^{(3)}_{16}(t) + Q^{(4)}_{17}(t) + Q^{(4,12)}_{12}(t)\$\pi_2(t)$

$+ Q^{(3,12)}_{12}(t)\$\pi_2(t) + Q^{(4,12)}_{11}(t)\$\pi_1(t) + Q^{(3,12)}_{11}(t)\$\pi_1(t)$

$\pi_2(t) = Q_{29}(t) + Q^{(12)}_{21}(t)\$\pi_1(t) + Q^{(12)}_{22}(t)\$\pi_2(t)$ (35-37)

Taking Laplace Stieltjes transform of relations and solving for $\tilde{\pi}_0(s)$ (35-37), weget

$$\tilde{\pi}_0(s) = N_1(s)/\ D_1(s) \quad (38)$$

where

$N_1(s) = (\tilde{Q}_{15} + \tilde{Q}^{(3)}_{16} + \tilde{Q}^{(4)}_{17})[\tilde{Q}_{01}(1 - \tilde{Q}^{(12)}_{22}) + \tilde{Q}_{02}\tilde{Q}^{(12)}_{21}]$

$+ \tilde{Q}_{29}[\tilde{Q}_{01}(\tilde{Q}^{(4,12)}_{12} + \tilde{Q}^{(3,12)}_{12}) + \tilde{Q}_{02}(1 - \tilde{Q}^{(4,12)}_{11} - \tilde{Q}^{(3,12)}_{11})]$ (39)

and

$$D_1(s) = (1 - \tilde{Q}^{(4,12)}_{11} - \tilde{Q}^{(3,12)}_{11})(1 - \tilde{Q}^{(12)}_{22})$$

$$- \tilde{Q}^{(12)}_{21}(\tilde{Q}^{(4,12)}_{12} + \tilde{Q}^{(3,12)}_{12}) \quad (40)$$

By taking the limit $s \to 0$ in equation (81), one gets $\tilde{\pi}_0(0) = 1$, which implies that $\tilde{\pi}_0(t)$ is a proper distribution function.

$$E(T) = - \frac{d}{ds} \pi_0(s)|_{s=0} = \frac{D'_1(0) - N'_1(0)}{D_1(0)} = N_1/D_1 \quad (41)$$

where,

$$N_1 = [p^{(12)}_{21}(p_{15} + p^{(3)}_{16} + p^{(4)}_{17}) + p_{29}(1 - p^{(4,12)}_{11} - p^{(3,12)}_{11})]\mu_0$$
$$+ (p^{(12)}_{21} + p_{01}p_{29})[m_{15} + m^{(3)}_{16} + m^{(4)}_{17} + m^{(4,12)}_{12} + m^{(3,12)}_{12}$$
$$+ m^{(3,12)}_{11} + m^{(4,12)}_{11}] + [p^{(4,12)}_{12} + p^{(3,12)}_{12} + p_{02}(p_{15} + p^{(3)}_{16}$$
$$+ p^{(4)}_{17})][m_{29} + m^{(12)}_{21} + + m^{(12)}_{22}] \quad (42)$$

and

$$D_1 = (1 - p^{(4,12)}_{11} - p^{(3,12)}_{11})(1 - p^{(12)}_{22}) - p^{(12)}_{21}(p^{(4,12)}_{12} + p^{(3,12)}_{12}) \quad (43)$$

AVAILABILITY ANALYSIS

System availability is defined as

$A_i(t)$ = Pr[Starting from state S_i the system is available at epoch t without passing through any regenerative state] and

$M_i(t)$ = Pr[Starting from up state S_i the system remains up till epoch t without passing through any regenerative up state]

Thus,

$$M_0(t) = \bar{F}_1(t)\bar{F}_2(t)\bar{K}(t)$$
$$M_1(t) = \bar{F}_1(t)[\bar{W}(t).\bar{H}(t) + {}_0\!\int^t \bar{H}(u)\, dW(u)\bar{G}_1(t-u)$$
$$+ {}_0\!\int^t \bar{W}(u)dH(u)\bar{G}_2(t-u)]$$
$$+ [{}_0\!\int^t {}_0\!\int^w dW(u)\bar{H}(u)dG_1(w-u)\bar{K}(t-w)]\bar{F}_1(t)$$
$$+ [{}_0\!\int^t {}_0\!\int^w dH(u)\, \bar{W}(u)\, dG_2(w-u)\, \bar{K}(t-w)]\bar{F}_1(t)$$
$$M_2(t) = M_8(t) = M_{11}(t) = \bar{F}_1(t) \quad (44\text{-}46)$$

Now, obtaining $A_i(t)$ by using elementary probability argument;

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t)$$
$$A_1(t) = M_1(t) + q^{(3,12)}_{11}(t)©A_1(t) + q^{(4,12)}_{11}(t)©A_1(t)$$
$$+ q^{(3,12)}_{12}(t)©A_2(t) + q^{(4,12)}_{12}(t)©A_2(t) + q^{(4,7)}_{18}(t)©A_8(t)$$
$$+ q^{(3,6)}_{1,11}(t)©A_{11}(t) + q_{15}(t)©A_5(t)$$
$$A_2(t) = M_2(t) + q^{(12)}_{21}(t)©A_1(t) + q_{29}(t)©A_9(t) + q^{(12)}_{22}(t)©A_2(t)$$
$$A_5(t) = q_{58}(t)©A_8(t)$$
$$A_8(t) = M_8(t) + q^{(12)}_{81}(t)©A_1(t) + q^{(12)}_{82}(t)©A_2(t) + q^{(7)}_{88}(t)©A_8(t)$$
$$A_9(t) = q^{(10,12)}_{91}(t)©A_1(t) + q_{92}(t)©A_2(t) + q^{(10,12)}_{92}(t)©A_2(t)$$
$$+ q^{(10,7)}_{98}(t)©A_8(t)$$
$$A_{11}(t) = M_{11}(t) + q^{(12)}_{11,1}(t)©A_1(t) + q^{(12)}_{11,2}(t)©A_2(t)$$
$$+ q^{(6)}_{11,11}(t)©A_{11}(t) \quad (47\text{-}53)$$

Taking Laplace transform of above equation (47-53), and solving for $A^*_i(s)$ we get,

$A^*_0(s) = [k_0M^*_0(s) + k_1M^*_1(s) + k_2M^*_2(s) + k_8M^*_8(s)$
$+ k_{11}M^*_{11}(s)]/k_0$ (54)

The steady-state availability of the system is

$A_0(\infty) = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s.\, A_0^*(s) = N_2(0)/D'_2(0) = N_2/D_2$ (55)

$N_2 = [(1 - p_{88})\{p_{92} + (1 - p_{22})\} - p_{29}p_{98}p_{82}\}M_1 + (1 - p_{88})$
$.[p_{12}(1 - p_{11,11}) + p_{1,11}p_{11,2} + p_{11}[p_{29}p_{92} - (1 - p_{22})\}]m$
$+ (1 - p_{11,11})(p_{15}p_{58} + p_{18})[p_{82} + p_{29}p_{92} - (1 - p_{22})]m$
$+ p_{29}p_{98}[p_{1,11}(p_{82} + p_{11,2}) - p_{12}(1 - p_{11,11})]m$ (56)

and

$D_2 = [(1 - p_{88})p_{92} + (1 - p_{22}) - p_{29}p_{98}p_{82}][m_{11} + m_{12} + m_{15}$
$+ m^{(4,7)}{}_{18} + m^{(3,6)1,11}) + [(1 - p_{88})\{p_{12}(1 - p_{11,11}) + p_{1,11}p_{11,2}\}$
$+ p_{82}(1 - p_{11,11})(p_{15}p_{58} + p_{18})][m_{29} + m^{(12)}{}_{21} + m^{(12)}{}_{22}]$
$+ p_{15}(1 - p_{11,11})[p_{29}(p_{91}p_{82} - p_{81}p_{92}) + p_{82}p_{21}$
$+ p_{81}(1 - p_{22})]\mu_5 + (1 - p_{11,11})(p_{15}p_{58} + p_{18})\{p_{29}p_{92}$
$- (1 - p_{22})\} - p_{29}p_{98}\{p_{12}(1 - p_{11,11}) + p_{1,11}p_{11,2}\}]$
$.[m^{(12)}{}_{18} + m^{(12)}{}_{82} + m^{(7)}{}_{88}] + p_{29}[(1 - p_{11,11})\{p_{12}(1 - p_{88})$
$+ p_{82}(p_{15}p_{58} + p_{18})\} + p_{1,11}p_{11,2}(1 - p_{88})][m^{(10,12)}{}_{91} + m_{92}$
$+ m^{(10,12)}{}_{92} + m^{(10,7)}{}_{98}] + [p_{11}(1 - p_{88})p_{29}p_{92}$
$- p_{11}(1 - p_{22})(1 - p_{88}) + p_{29}p_{98}p_{82}p_{1,11}][m^{(12)}{}_{11,1} + m^{(12)}{}_{11,2}$
$+ m^{(6)}{}_{11,11})]$ (57)

BUSY PERIOD ANANLYSIS

(a) Let $B_i(t)$ is the probability that the system is under repair by ordinary repair facility at time t, Thus the following recursive relations among $B_i(t)$'s can be obtained as ;

$B_0(t) = q_{01}(t)©B_1(t) + q_{02}(t)©B_2(t)$
$B_1(t) = W_1(t) + q^{(3,12)}{}_{11}(t)©B_1(t) + q^{(4,12)}{}_{11}(t)©B_1(t)$
$+ q^{(3,12)}{}_{12}(t)©B_2(t) + q^{(4,12)}{}_{12}(t)©B_2(t) + q^{(4,7)}{}_{18}(t)©B_8(t)$
$+ q^{(3,6)}{}_{1,11}(t)©B_{11}(t) + q_{15}(t)©B_5(t)$
$B_2(t) = q^{(12)}{}_{21}(t)©B_1(t) + q_{29}(t)©B_9(t) + q^{(12)}{}_{22}(t)©B_2(t)$
$B_5(t) = q_{58}(t)©B_8(t)$
$B_8(t) = q^{(12)}{}_{81}(t)©B_1(t) + q^{(12)}{}_{82}(t)©B_2(t) + q^{(7)}{}_{88}(t)©B_8(t)$
$B_9(t) = q^{(10,12)}{}_{91}(t)©B_1(t) + q_{92}(t)©B_2(t) + q^{(10,12)}{}_{92}(t)©B_2(t)$
$+ q^{(10,7)}{}_{98}(t)©B_8(t)$
$B_{11}(t) = W_{11}(t) + q^{(12)}{}_{11,1}(t)©B_1(t) + q^{(12)}{}_{11,2}(t)©B_2(t)$
$+ q^{(6)}{}_{11,11}(t)©B_{11}(t)$ (58-64)

Taking Laplace transform of above equations (58-64) and solving for $B^*_0(s)$, we get,

$B_0 = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} s\, B^*(s) = N_3(0)/D'_3(0) = N_3/D_3$ (65)

$N_3 = [(1 - p_{88})\{p_{92} + (1 - p_{22})\} - p_{29}p_{98}p_{82}]w_1$
$+ [p_{11}(1 - p_{88})\{p_{29}p_{92} - (1 - p_{22})\} + p_{29}p_{98}p_{82}p_{11,11}]n_1$ (66)

and D_3 is same as D_2 in (57).

(b) Let $R_i(t)$ is the probability that the system is under repair by ordinary repair facility at time t, Thus the following recursive relations among $R_i(t)$'s can be obtained as ;

$R_0(t) = q_{01}(t)©R_1(t) + q_{02}(t)©R_2(t)$
$R_1(t) = W_1(t) + q^{(3,12)}{}_{11}(t)©R_1(t) + q^{(4,12)}{}_{11}(t)©R_1(t)$
$+ q^{(3,12)}{}_{12}(t)©R_2(t) + q^{(4,12)}{}_{12}(t)©R_2(t) + q^{(4,7)}{}_{18}(t)©R_8(t)$
$+ q^{(3,6)}{}_{1,11}(t)©R_{11}(t) + q_{15}(t)©R_5(t)$
$R_2(t) = q^{(12)}{}_{21}(t)©R_1(t) + q_{29}(t)©R_9(t) + q^{(12)}{}_{22}(t)©R_2(t)$
$R_5(t) = W_5(t) + q_{58}(t)©R_8(t)$
$R_8(t) = W_8(t) + q^{(12)}{}_{81}(t)©R_1(t) + q^{(12)}{}_{82}(t)©R_2(t) + q^{(7)}{}_{88}(t)©R_8(t)$

$R_9(t) = W_9(t) + q^{(10,12)}_{91}(t)©R_1(t) + q_{92}(t)©R_2(t)$
$+ q^{(10,12)}_{92}(t)©R_2(t) + q^{(10,7)}_{98}(t)©R_8(t)$
$R_{11}(t) = q^{(12)}_{11,1}(t)©R_1(t) + q^{(12)}_{11,2}(t)©R_2(t)$
$+ q^{(6)}_{11,11}(t)©R_{11}(t)$ (67-73)

Now, taking Laplace transform of above equations (67-73), and solving for $R^*_0(s)$, we get,
$R_0 = \lim_{t\to\infty} R_0(t) = \lim_{s\to 0} s\,R^*(s) = N_4(0)/D'_4(0) = N_4/D_4$ (74)

$N_4 = [(1 - p_{88})\{p_{92} + (1 - p_{22})\} - p_{29}p_{98}p_{82}]w_1 + p_{15}(1 - p_{11,11})$
$.[(1 - p_{22})(1 - p_{88}) + p_{29}[p_{98}p_{82} + p_{92}(1 - p_{88})\}]n_2$
$+ [(1 - p_{11,11})(p_{15}p_{58} + p_{18})\{p_{29}p_{92} - (1 - p_{22})$
$- p_{29}p_{98}\{p_{12}(1 - p_{11,11}) + p_{11,11}p_{11,2}\}]n_2 + [(1 - p_{11,11})$
$.\{p_{12}(1 - p_{88}) + p_{82}(p_{15}p_{58} + p_{18})\} + p_{11,11}p_{11,2}(1 - p_{88})]p_{29}w_9$ (75)
and D_4 is same as D_2 in (57).

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