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## ORIGINAL ARTICLE

## Stochastic Analysis of a Priority Unit System with Final Trial of the Repaired Unit

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#### Abstract

The present paper deals with the Stochastic Analysis of a Priority Unit System in which first unit is of higher cost and production capacity so it is considered as priority unit. The priority unit gets priority for repair, final trial, post repair and operation if both the units are in Normal operative mode. Also after each repair the repaired unit is sent for final trial to test whether the repaired unit is working properly with its full efficiency or not. If it is found to be inefficient then send it for post repair. Key words: Stochastic Analysis, Priority Unit System, Final Trial, Repaired Unit Received: 10th July, 2016, Revised: 25 th August. 2016, Accepted: $27^{\text {th }}$ August. 2016 ©2016 Council of Research \& Sustainable Development, India How to cite this article: Singh S.V. (2016): Stochastic Analysis of a Priority Unit System with Final Trial of the Repaired Unit. Annals of Natural Sciences, Vol. 2[3]: September, 2016: 27-34.


## INTRODUCTION

The purpose of this paper is to present a two unit cold standby system in which first unit is of higher cost and production capacity so it is considered as priority unit. The priority unit gets priority for repair, final trial, post repair and operation if both the units are in Normal operative mode. Also after each repair the repaired unit is sent for final trial to test whether the repaired unit is working properly with its full efficiency or not. If it is found to be inefficient then send it for post repair. Using regenerative point technique with Markov renewal process, the following reliability characteristics of interest are obtained.

## MODEL DESCRIPTION AND ASSUMPTIONS

1. The system consists of two units which are non-identical. Initially first unit is operative and the second is kept as cold standby.
2. Whenever first unit fails the second unit becomes operative instantaneously by an automatic transfer switch, which is always perfect.
3. The first unit is of very high cost and production capacity so it is treated as priority unit and the second unit is treated as non-priority unit.
4. After each repair of failed unit, the repaired unit is sent for "final trial" in which the unit is kept as operative for a fixed amount of time is known as "critical time". If the repaired unit operates successfully with its full efficiency without any failure then send it for operation or cold standby otherwise for post repair. The probability that the repaired unit will transferred as operative after final trial is fixed.
5. The first unit gets priority in operation (when both the units are alive), repair, final trial and post repair.
6. A single repair facility is considered in the system for repair, final trial and post repair.
7. Failure time distributions of first and second units are exponential with different failure rates, while the distributions of completing repair, final trial and post repair are assumed to be general.

## NOTATION AND SYMBOLS

Fig. 1: Transitions between the various states


| $\mathrm{N}_{10}$ | Normal Ist unit kept as operative |
| :---: | :---: |
| $\mathrm{N}_{20}$ | Normal IInd unit kept as operative |
| $\mathrm{N}_{2 \mathrm{CS}}$ | Normal IInd unit kept as cold standby |
| $\mathrm{F}_{1 \mathrm{r}}$ | Failed I ${ }^{\text {st }}$ unit under repair |
| $\mathrm{F}_{2} \mathrm{r}$ | Failed II ${ }^{\text {ndt }}$ unit under repair |
| $\mathrm{F}_{1 \mathrm{ft}}$ | Repaired Ist unit under final trial |
| $\mathrm{F}_{2 \mathrm{ft}}$ | Repaired IInd unit under final trial |
| $\mathrm{F}_{1 \mathrm{pr}}$ | Failed $\mathrm{I}^{\text {st }}$ unit under post repair |
| $\mathrm{F}_{2 \mathrm{pr}}$ | Failed II ${ }^{\text {nd }}$ unit under post repair |
| $\mathrm{F}_{2 \mathrm{wr}}$ | Failed II ${ }^{\text {nd }}$ unit is waiting for repair |
| $\mathrm{F}_{2 \text { wft }}$ | Repaired II ${ }^{\text {nd }}$ unit waiting for final trial |
| $\mathrm{F}_{2 \mathrm{wpr}}$ | Failed II ${ }^{\text {nd }}$ unit waiting for post repair |
| $\alpha$ | Constant failure rate of Ist unit |
| $\beta$ | Constant failure rate of IInd unit |
| $\mathrm{f}_{\mathrm{i}}(),. \mathrm{F}_{\mathrm{i}}($.$) :$ | pdf and cdf of time to complete repair of $\mathrm{ith}(\mathrm{i}=1,2)$ failed unit |
| $\mathrm{g}_{\mathrm{i}}(),. \mathrm{G}_{\mathrm{i}}($.$) :$ | pdf and cdf of time to complete final trial $\mathrm{i}^{\text {th }}(\mathrm{i}=1,2)$ repaired unit |
| $\mathrm{h}_{\mathrm{i}}(),. \mathrm{H}_{\mathrm{i}}($.$) :$ | pdf and cdf of time to complete post repair of $\mathrm{i}^{\text {th }}(\mathrm{i}=1,2)$ failed unit |
| p : | Probability that the repaired unit will be transferred as operating unit after completing "final trial" |
| q | Probability that the repaired unit will be sent for post repair after completing "final trial" |

$\mathrm{m}_{1} \quad$ : Mean time for completing repair
$m_{2} \quad$ : Mean time for completing final trial
$m_{3} \quad$ : Mean time for completing post repair
Using the above notation and symbols the possible states of the system are

## Up States

$\mathrm{S}_{0} \equiv\left(\mathrm{~N}_{10}, \mathrm{~N}_{2 \mathrm{CS}}\right) \quad \mathrm{S}_{1} \equiv\left(\mathrm{~F}_{1 \mathrm{r}}, \mathrm{N}_{20}\right) \quad \mathrm{S}_{2} \equiv\left(\mathrm{~F}_{1 \mathrm{ft}}, \mathrm{N}_{20}\right)$
$\mathrm{S}_{3} \equiv\left(\mathrm{~F}_{1 \mathrm{pr}}, \mathrm{N}_{20}\right) \quad \mathrm{S}_{7} \equiv\left(\mathrm{~N}_{10}, \mathrm{~F}_{2 \mathrm{r}}\right) \quad \mathrm{S}_{8} \equiv\left(\mathrm{~N}_{10}, \mathrm{~F}_{2 \mathrm{ft}}\right)$
$\mathrm{S}_{9} \equiv\left(\mathrm{~N}_{10}, \mathrm{~F}_{2 \mathrm{pr}}\right)$

## Down States

$\mathrm{S}_{4} \equiv\left(\mathrm{~F}_{1 \mathrm{r}}, \mathrm{F}_{2 \mathrm{wr}}\right) \quad \mathrm{S}_{5} \equiv\left(\mathrm{~F}_{1 \mathrm{ft}}, \mathrm{F}_{2 \mathrm{wr}}\right) \quad \mathrm{S}_{6} \equiv\left(\mathrm{~F}_{1 \mathrm{pr}}, \mathrm{F}_{2 \mathrm{wr}}\right)$
$\mathrm{S}_{10} \equiv\left(\mathrm{~F}_{1 \mathrm{r}}, \mathrm{F}_{2 \mathrm{wft}}\right) \quad \mathrm{S}_{11} \equiv\left(\mathrm{~F}_{1 \mathrm{ft}}, \mathrm{F}_{2 \mathrm{wft}}\right) \quad \mathrm{S}_{12} \equiv\left(\mathrm{~F}_{1 \mathrm{pr}}, \mathrm{F}_{2 \mathrm{wft}}\right)$
$\mathrm{S}_{13} \equiv\left(\mathrm{~F}_{1 \mathrm{r}}, \mathrm{F}_{2 \mathrm{wpr}}\right) \quad \mathrm{S}_{14} \equiv\left(\mathrm{~F}_{1 \mathrm{ft}}, \mathrm{F}_{2 \mathrm{wpr}}\right) \quad \mathrm{S}_{15} \equiv\left(\mathrm{~F}_{1 \mathrm{pr}}, \mathrm{F}_{2 \mathrm{wpr}}\right)$

## TRANSITION PROBABILITIES

Let $T_{0}(=0), T_{1}, T_{2}$ denotes the entry into any state $S_{i} \in E$. Let $X_{n}$ be the states visited at epoch $T_{n+1}$ i.e. just after the transition at $T_{n}$. Then $\left\{T_{n}, X_{n}\right\}$ is a Markov-renewal process with state space $E$ and is semi Markov-Kernel over $E$.
$\mathrm{Q}_{\mathrm{ij}}(\mathrm{t})=\operatorname{Pr}\left[\mathrm{X}_{\mathrm{n}+1}=\mathrm{S}_{\mathrm{j}}, \mathrm{T}_{\mathrm{n}+1}-\mathrm{T}_{\mathrm{n}} \leq \mathrm{t} \mid \mathrm{X}_{\mathrm{n}}=\mathrm{S}_{\mathrm{i}}\right]$
The stochastic matrix of the embedded Markov chain is
$\mathrm{P}=\left(\mathrm{p}_{\mathrm{ij}}\right)=\mathrm{Q}_{\mathrm{ij}}(\infty)=\mathrm{Q}(\infty)$.
The Non-Zero Elements Of $P_{i j}$ Are Given Below:

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\(\mathrm{p}_{12}=\mathrm{f}^{*}{ }_{1}(\beta)\)
\(\mathrm{p}_{20}=\mathrm{p} . \mathrm{g}^{*}{ }_{1}(\beta)\)
\(\mathrm{p}_{25}=1-\mathrm{g}_{1}(\beta)\)
\(p^{(5)}{ }_{27}=p \cdot\left[1-g_{1}^{*}(\beta)\right]\)
    \(\mathrm{p}_{14}=1-\mathrm{f}^{*}{ }_{1}(\beta)=\mathrm{p}^{(4)_{15}}\)
    \(\mathrm{p}_{23}=\mathrm{q} \cdot \mathrm{g}^{*}{ }_{1}(\beta)\)
    \(\mathrm{p}^{(5)}{ }_{26}=\mathrm{q} \cdot\left[1-\mathrm{g}^{*}(\beta)\right]\)
\(\mathrm{p}_{30}=\mathrm{h}^{*}{ }_{1}(\beta)\)
\(p_{36}=1-h^{*}{ }_{1}(\beta)=p^{(6)}{ }_{37}\)
    \(\mathrm{p}_{56}=\mathrm{p}_{11,12}=\mathrm{p}_{14,15}=\mathrm{q}\)
\(\mathrm{p}_{57}=\mathrm{p}_{11,8}=\mathrm{p}_{14,9}=\mathrm{p}\)
    \(\mathrm{p}_{74}=1-\mathrm{f}_{2}{ }_{2}(\alpha)\)
\(\mathrm{p}_{78}=\mathrm{f}_{2}(\alpha) \mathrm{p}_{80}=\mathrm{p} \cdot \mathrm{g}^{*}(\alpha)\)
    \(\mathrm{p}_{89}=\mathrm{q} \cdot \mathrm{g}_{2}^{*}(\alpha)\)
\(\mathrm{p}_{8,10}=1-\mathrm{g}^{*}{ }_{2}(\alpha)\)
    \(\mathrm{p}_{90}=\mathrm{h}^{*}{ }_{2}(\alpha)\)
\(\mathrm{p}_{9,13}=1-\mathrm{h}^{*}(\alpha)\)
\(\mathrm{p}_{01}=\mathrm{p}_{45}=\mathrm{p}_{67}=\mathrm{p}_{10,11}=\mathrm{p}_{12,8}=\mathrm{p}_{13,14}=\mathrm{p}_{15,9}=1\)
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From the above probabilities the following relation can be easily verifies as;
$\mathrm{p}_{01}=1 \quad \mathrm{p}_{12}+\mathrm{p}_{14}=1=\mathrm{p}_{12}+\mathrm{p}^{(4)}{ }_{15}$
$\mathrm{p}_{20}+\mathrm{p}_{23}+\mathrm{p}_{25}=1=\mathrm{p}_{20}+\mathrm{p}_{23}+\mathrm{p}^{(5)}{ }_{26}+\mathrm{p}^{(5)}{ }_{27}$
$\mathrm{p}_{30}+\mathrm{p}_{36}=1=\mathrm{p}_{30}+\mathrm{p}^{(6)} 37 \quad \mathrm{p}_{45}=1$
$\mathrm{p}_{56}+\mathrm{p}_{57}=1 \quad \mathrm{p}_{67}=1$
$\mathrm{p}_{74}+\mathrm{p}_{78}=1 \quad \mathrm{p}_{80}+\mathrm{p}_{89}+\mathrm{p}_{8,10}=1$
$\mathrm{p}_{90}+\mathrm{p}_{9,13}=1 \quad \mathrm{p}_{10,11}=1$
$\mathrm{p}_{11,8}+\mathrm{p}_{11,12}=1 \quad \mathrm{p}_{12,8}=1$
$\mathrm{p}_{13,14}=1 \quad \mathrm{p}_{14,9}+\mathrm{p}_{14,15}=1$
$\mathrm{p}_{15,9}=1$

## MEAN SOJOURN TIMES

The mean sojourn time in a state $S_{i}$ is defined as the length of stay in time in a state $S_{i}$ before transiting to any other state.
If T denotes the sojourn time in $\mathrm{S}_{\mathrm{i}}$, then

$$
\begin{equation*}
\mu_{\mathrm{i}}=\mathrm{E}(\mathrm{~T})=\int_{0}^{\infty} \mathrm{P}_{\mathrm{r}}[\mathrm{~T}>\mathrm{t}] \mathrm{dt} \tag{37}
\end{equation*}
$$

Using This We Can Obtain The Following Expressions:
$\mu_{0}=\frac{1}{\alpha} \quad \mu_{1}=\frac{1}{\beta}\left\{1-\mathrm{f}_{1}{ }^{*}(\beta)\right\} \quad \mu_{2}=\frac{1}{\beta}\left\{1-\mathrm{g}_{1}{ }^{*}(\beta)\right\}$

$$
\begin{array}{ll}
\mu_{3}=\frac{1}{\beta}\left\{1-\mathrm{h}_{1}{ }^{*}(\beta)\right\} \quad \mu_{7}=\frac{1}{\alpha}\left\{1-\mathrm{f}_{2}{ }^{*}(\alpha)\right\} \quad \mu_{8}=\frac{1}{\alpha}\left\{1-\mathrm{g}_{2}{ }^{*}(\alpha)\right\} \\
\mu_{9}=\frac{1}{\alpha}\left\{1-\mathrm{h}_{2}{ }^{*}(\alpha)\right\} & \tag{38-44}
\end{array}
$$

## MEAN TIME TO SYSTEM FAILURE (MTSF)

the distribution function $\pi_{\mathrm{i}}(\mathrm{t})$ of the time to system failure with starting state $\mathrm{S}_{0}$.

$$
\begin{align*}
& \pi_{0}(\mathrm{t})=\mathrm{Q}_{01}(\mathrm{t}) \$ \pi_{1}(\mathrm{t}) \\
& \pi_{1}(\mathrm{t})=\mathrm{Q}_{12}(\mathrm{t}) \$ \pi_{2}(\mathrm{t})+\mathrm{Q}_{14}(\mathrm{t}) \\
& \pi_{2}(\mathrm{t})=\mathrm{Q}_{20}(\mathrm{t}) \$ \pi_{0}(\mathrm{t})+\mathrm{Q}_{23}(\mathrm{t}) \$ \pi_{3}(\mathrm{t})+\mathrm{Q}_{25}(\mathrm{t}) \\
& \pi_{3}(\mathrm{t})=\mathrm{Q}_{30}(\mathrm{t}) \$ \pi_{0}(\mathrm{t})+\mathrm{Q}_{36}(\mathrm{t}) \\
& \pi_{7}(\mathrm{t})=\mathrm{Q}_{74}(\mathrm{t})+\mathrm{Q}_{78}(\mathrm{t}) \$ \pi_{8}(\mathrm{t}) \\
& \pi_{8}(\mathrm{t})=\mathrm{Q}_{80}(\mathrm{t}) \$ \pi_{0}(\mathrm{t})+\mathrm{Q}_{89}(\mathrm{t}) \$ \pi_{9}(\mathrm{t})+\mathrm{Q}_{8,10}(\mathrm{t}) \\
& \pi_{9}(\mathrm{t})=\mathrm{Q}_{90}(\mathrm{t}) \$ \pi_{0}(\mathrm{t})+\mathrm{Q}_{9,13}(\mathrm{t}) \tag{45-51}
\end{align*}
$$

Taking Laplace Stieltjes transform of relations and solving for $\pi_{0}(\mathrm{~s})$, we get

$$
\begin{equation*}
\pi_{0}(\mathrm{~s})=\mathrm{N}_{1}(\mathrm{~s}) / \mathrm{D}_{1}(\mathrm{~s}) \tag{52}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{N}_{1}(\mathrm{~s})=\tilde{\mathcal{Q}}_{01} \tilde{Q}_{14}+\tilde{\mathcal{Q}}_{01} \tilde{Q}_{12}\left(\tilde{\sim}_{25}+\tilde{Q}_{23} \tilde{Q}_{36}\right) \tag{53}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{D}_{1}(\mathrm{~s})=1-\tilde{Q}_{01} \tilde{Q}_{12}\left(\tilde{Q}_{20}+\tilde{Q}_{23} \tilde{Q}_{30}\right) \tag{54}
\end{equation*}
$$

By taking the limits $\rightarrow 0$ in equation (154), one gets $\tilde{\pi}_{0}(0)=1$, which implies that $\tilde{\pi}_{0}(\mathrm{t})$ is a proper distribution function.
$E(T)=-\left.\frac{d}{d s} \pi_{0}(s)\right|_{s=0}=\frac{\mathrm{D}^{\prime}(0)-\mathrm{N}_{1}(0)}{\mathrm{D}_{1}(0)}=N_{1} / D_{1}$
where
$\mathrm{N}_{1}=\mu_{0}+\mu_{1}+\mathrm{p}_{12}\left(\mu_{2}+\mathrm{p}_{23} \mu_{3}\right)$
and
$\mathrm{D}_{1}=1-\mathrm{p}_{12}\left(\mathrm{p}_{20}+\mathrm{p}_{23} \mathrm{p}_{30}\right)$

## AVAILABILITY ANALYSIS:

System availability is defined as
$\mathrm{A}_{\mathrm{i}}(\mathrm{t})=\operatorname{Pr}$ [Starting from state $\mathrm{S}_{\mathrm{i}}$ the system is available at epoch t without passing through any regenerative state] and
$M_{i}(\mathrm{t})=\operatorname{Pr}$ [Starting from upstate $\mathrm{S}_{\mathrm{i}}$ the system remains up till epoch t without passing through any regenerative up state]
Thus
$\mathrm{M}_{0}(\mathrm{t})=\mathrm{e}^{-\alpha_{\mathrm{t}}} \quad \mathrm{M}_{1}(\mathrm{t})=\mathrm{e}^{-\beta_{\mathrm{t}}} \cdot \overline{\mathrm{F}}_{1}(\mathrm{t}) \quad \mathrm{M}_{2}(\mathrm{t})=\mathrm{e}^{-\beta_{\mathrm{t}}} . \bar{G}_{1}(\mathrm{t})$
$\mathrm{M}_{3}(\mathrm{t})=\mathrm{e}^{-\mathrm{B}_{\mathrm{t}}} \cdot \bar{H}_{1}(\mathrm{t})$
$M_{7}(\mathrm{t})=\mathrm{e}^{-\alpha_{\mathrm{t}}} . \overline{\mathrm{F}}_{2}(\mathrm{t})$
$\mathrm{M}_{8}(\mathrm{t})=\mathrm{e}^{-\alpha_{\mathrm{t}}} \cdot \bar{G}_{2}(\mathrm{t})$
$\mathrm{M}_{9}(\mathrm{t})=\mathrm{e}^{-\alpha_{\mathrm{t}}} \cdot \bar{H}_{2}(\mathrm{t})$
(58-64)

Now, obtaining $\mathrm{A}_{\mathrm{i}}(\mathrm{t})$ by using elementary probability argument;
$\mathrm{A}_{0}(\mathrm{t})=\mathrm{M}_{0}(\mathrm{t})+\mathrm{q}_{01}(\mathrm{t}) \subseteq \mathrm{A}_{1}(\mathrm{t})$
$\mathrm{A}_{1}(\mathrm{t})=\mathrm{M}_{1}(\mathrm{t})+\mathrm{q}_{12}(\mathrm{t}) \odot \mathrm{A}_{2}(\mathrm{t})+\mathrm{q}^{(4)}{ }_{15}(\mathrm{t}) \odot \mathrm{A}_{5}(\mathrm{t})$
$\mathrm{A}_{2}(\mathrm{t})=\mathrm{M}_{2}(\mathrm{t})+\mathrm{q}_{20}(\mathrm{t}) \odot \mathrm{A}_{0}(\mathrm{t})+\mathrm{q}_{23}(\mathrm{t}) \odot \mathrm{A}_{3}(\mathrm{t})+\mathrm{q}{ }^{(5)}{ }_{26}(\mathrm{t}) \odot \mathrm{A}_{6}(\mathrm{t})+\mathrm{q}{ }^{(5)}{ }_{27}(\mathrm{t}) \odot \mathrm{A}_{7}(\mathrm{t})$
$\mathrm{A}_{3}(\mathrm{t})=\mathrm{M}_{3}(\mathrm{t})+\mathrm{q}_{30}(\mathrm{t}) \odot \mathrm{A}_{0}(\mathrm{t})+\mathrm{q}^{(7)}{ }_{37}(\mathrm{t}) \odot \mathrm{A}_{7}(\mathrm{t})$
$\mathrm{A}_{4}(\mathrm{t})=\mathrm{q}_{45}(\mathrm{t}) \odot \mathrm{A}_{5}(\mathrm{t})$
$\mathrm{A}_{5}(\mathrm{t})=\mathrm{q}_{56}(\mathrm{t}) \odot \mathrm{A}_{6}(\mathrm{t})+\mathrm{q}_{57}(\mathrm{t}) \odot \mathrm{A}_{7}(\mathrm{t})$
$A_{6}(t)=q_{67}(t) ® A_{7}(t)$
$\mathrm{A}_{7}(\mathrm{t})=\mathrm{M}_{7}(\mathrm{t})+\mathrm{q}_{74}(\mathrm{t}) \odot \mathrm{A}_{4}(\mathrm{t})+\mathrm{q}_{78}(\mathrm{t}) \bigcirc \mathrm{A}_{8}(\mathrm{t})$
$\mathrm{A}_{8}(\mathrm{t})=\mathrm{M}_{8}(\mathrm{t})+\mathrm{q}_{80}(\mathrm{t}) \bigcirc \mathrm{A}_{0}(\mathrm{t})+\mathrm{q}_{89}(\mathrm{t}) \subseteq \mathrm{A}_{9}(\mathrm{t})+\mathrm{q}_{8,10}(\mathrm{t}) \subseteq \mathrm{A}_{10}(\mathrm{t})$
$\mathrm{A}_{9}(\mathrm{t})=\mathrm{M}_{9}(\mathrm{t})+\mathrm{q}_{90}(\mathrm{t}) \odot \mathrm{A}_{0}(\mathrm{t})+\mathrm{q}_{9,13}(\mathrm{t}) \odot \mathrm{A}_{13}(\mathrm{t})$
$\mathrm{A}_{10}(\mathrm{t})=\mathrm{q}_{10,11}(\mathrm{t}) \odot \mathrm{A}_{11}(\mathrm{t})$
$\mathrm{A}_{11}(\mathrm{t})=\mathrm{q}_{11,8}(\mathrm{t}) \mathbb{C} \mathrm{A}_{8}(\mathrm{t})+\mathrm{q}_{11,12}(\mathrm{t}) \subseteq \mathrm{A}_{12}(\mathrm{t})$
$\mathrm{A}_{12}(\mathrm{t})=\mathrm{q}_{12,8}(\mathrm{t}) \odot \mathrm{A}_{8}(\mathrm{t})$
$\mathrm{A}_{13}(\mathrm{t})=\mathrm{q}_{13,14}(\mathrm{t}) \odot \mathrm{A}_{14}(\mathrm{t})$
$\mathrm{A}_{14}(\mathrm{t})=\mathrm{q}_{14,9}(\mathrm{t}) \odot \mathrm{A}_{9}(\mathrm{t})+\mathrm{q}_{14,15}(\mathrm{t}) \odot \mathrm{A}_{15}(\mathrm{t})$
$\mathrm{A}_{15}(\mathrm{t})=\mathrm{q}_{15,9}(\mathrm{t}) \odot \mathrm{A}_{9}(\mathrm{t})$
Now taking Laplace transform of above equation (65-80), and solving for pointwise availability $\mathrm{A}^{*}(\mathrm{~s})$, we get,

$$
\begin{equation*}
A^{*}(\mathrm{~s})=\frac{\mathrm{N}_{2}(\mathrm{~s})}{\mathrm{D}_{2}(\mathrm{~s})} \tag{81}
\end{equation*}
$$

Where in terms of

```
\(\mathrm{a}=1-\mathrm{q}^{*}{ }_{9,13} \mathrm{q}^{*}{ }_{13,14}\left(\mathrm{q}^{*}{ }_{14,9}+\mathrm{q}^{*}{ }_{14,15} \mathrm{q}^{*}{ }_{15,9}\right)\)
\(\mathrm{b}=1-\mathrm{q}^{*}{ }_{8,10} \mathrm{q}^{*}{ }_{10,11}\left(\mathrm{q}^{*}{ }_{11,8}+\mathrm{q}^{*}{ }_{11,12} \mathrm{q}^{*}{ }_{12,8}\right)\)
\(\mathrm{c}=\mathrm{q}^{*}{ }_{12}\left(\mathrm{q}^{*}{ }^{(5)}{ }_{27}+\mathrm{q}^{*}{ }_{23} \mathrm{q}^{*}(6)_{37}+\mathrm{q}^{*}(5)_{26} \mathrm{q}^{*}{ }_{67}\right)+\mathrm{q}^{*}(4)_{15}\left(\mathrm{q}^{*}{ }_{57}+\mathrm{q}^{*}{ }_{56} \mathrm{q}^{*}{ }_{67}\right)\)
\(\mathrm{d}=1-\mathrm{q}^{*}{ }_{74} \mathrm{q}^{*}{ }_{45}\left(\mathrm{q}^{*}{ }_{57}+\mathrm{q}^{*}{ }_{56} \mathrm{q}^{*}{ }_{67}\right)\)
```

We have
$\mathrm{N}_{2}(\mathrm{~s})=\left[\mathrm{M}_{0}^{*}+\mathrm{q}^{*}{ }_{01}\left\{\mathrm{M}_{1}{ }_{1}+\mathrm{q}^{*}{ }_{12}\left(\mathrm{M}_{2}{ }_{2}+\mathrm{q}^{*}{ }_{23} \mathrm{M}^{*} 3\right)\right\}\right]$ a.b.d $+\mathrm{q}^{*}{ }_{01}\left[\mathrm{M}^{*}{ }_{7} \mathrm{a} \cdot \mathrm{b}+\mathrm{q}^{*}{ }_{78}\left\{\mathrm{M}^{*}{ }_{8} \cdot \mathrm{a}+\mathrm{q}^{*}-\right.\right.$ $\left.\left.{ }_{89} \mathrm{M}^{*} 9\right\}\right]$.c
and
$\left.\mathrm{D}_{2}(\mathrm{~s})=1-\mathrm{q}^{*}{ }_{01} \mathrm{q}^{*}{ }_{12}\left(\mathrm{q}^{*}{ }_{20}+\mathrm{q}^{*}{ }_{23} \mathrm{q}^{*}{ }_{30}\right)\right]$ a.b.d $-\mathrm{q}^{*}{ }_{01} \mathrm{q}^{*}{ }_{78}\left[\mathrm{q}^{*}{ }_{80} \cdot \mathrm{a}+\mathrm{q}^{*}{ }_{89} \mathrm{q}^{*}{ }_{90}\right] . \mathrm{c}$
By taking the limit $s \rightarrow 0$ in the relation (205), we get $\mathrm{D}_{2}(0)=0$, therefore the steady state availability of the system when it starts operations from $S_{0}$ is

$$
\begin{equation*}
\underset{\mathrm{t}^{\rightarrow \infty}}{\mathrm{A}_{0}(\infty)}=\lim _{\mathrm{s} \rightarrow 0} \mathrm{~A}_{0}(\mathrm{t})=\lim \mathrm{s} \cdot \mathrm{~A}_{0} *(\mathrm{~s})=\mathrm{N}_{2}(0) / \mathrm{D}_{2}^{\prime}(0)=\mathrm{N}_{2} / \mathrm{D}_{2} \tag{88}
\end{equation*}
$$

Where
$\mathrm{N}_{2}=\left[\mu_{0}+\left\{\mu_{1}+\mathrm{p}_{12}\left(\mu_{2}+\mathrm{p}_{23} \mu_{3}\right)\right\}\right]\left[\mathrm{p}_{78}\left(\mathrm{p}_{80}+\mathrm{p}_{89}\right) \mathrm{p}_{90}\right]+\left[\mu_{7}\left(\mathrm{p}_{80}+\mathrm{p}_{89}\right) \mathrm{p}_{90}+\mathrm{p}_{78}\left(\mu_{8} \mathrm{p}_{90}+\right.\right.$ $\left.\left.\mathrm{p}_{89} \mu_{9}\right)\right] \cdot\left[1-\mathrm{p}_{12}\left(\mathrm{p}_{20}+\mathrm{p}_{23} \mathrm{p}_{30}\right)\right]$
and

$$
\begin{align*}
\mathrm{D}_{2}= & \mathrm{p}_{78}\left(\mathrm{p}_{80}+\mathrm{p}_{89}\right) \mathrm{p}_{90}\left[\mu_{0}+\mathrm{m}_{1}+\mathrm{p}_{12}\left\{\mathrm{~m}_{2}+\mathrm{m}_{3}\left(1+\mathrm{p}^{(5)}{ }_{26}\right)\right\}+\mathrm{p}^{(4)}{ }_{15}\left(\mathrm{~m}_{2}\right.\right.  \tag{89}\\
& \left.\left.+\mathrm{p}_{56} \mathrm{~m}_{3}\right)\right]+\left[1-\mathrm{p}_{12}\left(\mathrm{p}_{20}+\mathrm{p}_{23} \mathrm{p}_{30}\right)\right]\left[( \mathrm { p } _ { 8 0 } + \mathrm { p } _ { 8 9 } ) \mathrm { p } _ { 9 0 } \left\{\mu _ { 7 } \mathrm { p } _ { 7 4 } \left(\mathrm{m}_{1}+\mathrm{m}_{2}\right.\right.\right. \\
& \left.\left.+\mathrm{p}_{56} \mathrm{~m}_{3}\right)\right\}+\mathrm{p}_{78} \mathrm{p}_{90}\left\{\mu_{8}+\mathrm{p}_{8,10}\left(\mathrm{~m}_{1}+\mathrm{m}_{2}+\mathrm{m}_{3}\right)\right\}+\mathrm{p}_{78} \mathrm{p}_{89}\left(\mu_{9}+\mathrm{m}_{1}\right. \\
& \left.\left.+\mathrm{p}_{9,13}\left(\mathrm{~m}_{2}+\mathrm{p}_{14,15} \mathrm{~m}_{3}\right)\right\}\right] \tag{90}
\end{align*}
$$

## BUSY PERIOD ANANLYSIS

Let us define $\mathrm{W}_{\mathrm{i}}(\mathrm{t})$ as the probability that the system is under repair by repair facility in state $S_{i} \varepsilon E$ at time $t$ without transiting to any regenerative state.

## Singh

Therefore
$\begin{array}{llll}\mathrm{W}_{1}(\mathrm{t})=\overline{\mathrm{F}}_{1}(\mathrm{t}) & \mathrm{W}_{2}(\mathrm{t})=\overline{\mathrm{G}}_{1}(\mathrm{t}) & \mathrm{W}_{3}(\mathrm{t})=\overline{\mathrm{H}}_{1}(\mathrm{t}) & \mathrm{W}_{4}(\mathrm{t})=\overline{\mathrm{F}}_{1}(\mathrm{t}) \\ \mathrm{W}_{5}(\mathrm{t})=\overline{\mathrm{G}}_{1}(\mathrm{t}) & \mathrm{W}_{6}(\mathrm{t})=\overline{\mathrm{H}}_{1}(\mathrm{t}) & \mathrm{W}_{7}(\mathrm{t})=\mathrm{e}^{-\alpha_{\mathrm{t}}} \overline{\mathrm{F}}_{2}(\mathrm{t}) & \mathrm{W}_{8}(\mathrm{t})=\mathrm{e}^{-\alpha_{t}} \overline{\mathrm{G}}_{2}(\mathrm{t}) \\ \mathrm{W}_{9}(\mathrm{t})=\mathrm{e}^{-\alpha_{\mathrm{t}}} \overline{\mathrm{H}}_{2}(\mathrm{t}) & \mathrm{W}_{10}(\mathrm{t})=\overline{\mathrm{F}}_{1}(\mathrm{t}) & \mathrm{W}_{11}(\mathrm{t})=\overline{\mathrm{G}}_{1}(\mathrm{t}) & \mathrm{W}_{12}(\mathrm{t})=\overline{\mathrm{H}}_{1}(\mathrm{t}) \\ \mathrm{W}_{13}(\mathrm{t})=\overline{\mathrm{F}}_{1}(\mathrm{t}) & \mathrm{W}_{14}(\mathrm{t})=\overline{\mathrm{G}}_{1}(\mathrm{t}) & \mathrm{W}_{15}(\mathrm{t})=\overline{\mathrm{H}}_{1}(\mathrm{t}) & \end{array}$
Also let $\mathrm{B}_{\mathrm{i}}(\mathrm{t})$ is the probability that the system is under repair by repair facility at time t , Thus the following recursive relations among $B_{i}(t)$ 's can be obtained as ;
$\mathrm{B}_{0}(\mathrm{t})=\mathrm{q}_{01}(\mathrm{t}) \bigcirc \mathrm{B}_{1}(\mathrm{t})$
$\mathrm{B}_{1}(\mathrm{t})=\mathrm{W}_{1}(\mathrm{t})+\mathrm{q}_{12}(\mathrm{t}) \odot \mathrm{B}_{2}(\mathrm{t})+\mathrm{q}^{(4)}{ }_{15}(\mathrm{t}) \odot \mathrm{B}_{5}(\mathrm{t})$
$\left.\mathrm{B}_{2}(\mathrm{t})=\mathrm{W}_{2}(\mathrm{t})+\mathrm{q}_{20}(\mathrm{t}) \odot \mathrm{B}_{0}(\mathrm{t})+\mathrm{q}_{23}(\mathrm{t}) \odot \mathrm{B}_{3}(\mathrm{t})+\mathrm{q}^{(5)}\right)_{26}(\mathrm{t}) \odot \mathrm{B}_{6}(\mathrm{t})+\mathrm{q}^{(5)_{27}}(\mathrm{t}) \odot \mathrm{B}_{7}(\mathrm{t})$
$\mathrm{B}_{3}(\mathrm{t})=\mathrm{W}_{3}(\mathrm{t})+\mathrm{q}_{30}(\mathrm{t}) \odot \mathrm{B}_{0}(\mathrm{t})+\mathrm{q}^{(7)_{37}}(\mathrm{t}) \odot \mathrm{B}_{7}(\mathrm{t})$
$\mathrm{B}_{4}(\mathrm{t})=\mathrm{W}_{4}(\mathrm{t})+\mathrm{q}_{45}(\mathrm{t}) \odot \mathrm{B}_{5}(\mathrm{t})$
$\mathrm{B}_{5}(\mathrm{t})=\mathrm{W}_{5}(\mathrm{t})+\mathrm{q}_{56}(\mathrm{t}) \odot \mathrm{B}_{6}(\mathrm{t})+\mathrm{q}_{57}(\mathrm{t}) \odot \mathrm{B}_{7}(\mathrm{t})$
$\mathrm{B}_{6}(\mathrm{t})=\mathrm{W}_{6}(\mathrm{t})+\mathrm{q}_{67}(\mathrm{t}) \odot \mathrm{B}_{7}(\mathrm{t})$
$\mathrm{B}_{7}(\mathrm{t})=\mathrm{W}_{7}(\mathrm{t})+\mathrm{q}_{74}(\mathrm{t}) \odot \mathrm{B}_{4}(\mathrm{t})+\mathrm{q}_{78}(\mathrm{t}) \odot \mathrm{B}_{8}(\mathrm{t})$
$\mathrm{B}_{8}(\mathrm{t})=\mathrm{W}_{8}(\mathrm{t})+\mathrm{q}_{80}(\mathrm{t}) \odot \mathrm{B}_{0}(\mathrm{t})+\mathrm{q}_{89}(\mathrm{t}) \odot \mathrm{B}_{9}(\mathrm{t})+\mathrm{q}_{8,10}(\mathrm{t}) \odot \mathrm{B}_{10}(\mathrm{t})$
$\mathrm{B}_{9}(\mathrm{t})=\mathrm{W}_{9}(\mathrm{t})+\mathrm{q}_{90}(\mathrm{t}) \odot \mathrm{B}_{0}(\mathrm{t})+\mathrm{q}_{9,13}(\mathrm{t}) \odot \mathrm{B}_{13}(\mathrm{t})$
$\mathrm{B}_{10}(\mathrm{t})=\mathrm{W}_{10}(\mathrm{t})+\mathrm{q}_{10,11}(\mathrm{t}) \odot \mathrm{B}_{11}(\mathrm{t})$
$\mathrm{B}_{11}(\mathrm{t})=\mathrm{W}_{11}(\mathrm{t})+\mathrm{q}_{11,8}(\mathrm{t}) \odot \mathrm{B}_{8}(\mathrm{t})+\mathrm{q}_{11,12}(\mathrm{t}) \odot \mathrm{B}_{12}(\mathrm{t})$
$\mathrm{B}_{12}(\mathrm{t})=\mathrm{W}_{12}(\mathrm{t})+\mathrm{q}_{12,8}(\mathrm{t}) \odot \mathrm{B}_{8}(\mathrm{t})$
$\mathrm{B}_{13}(\mathrm{t})=\mathrm{W}_{13}(\mathrm{t})+\mathrm{q}_{13,14}(\mathrm{t}) \odot \mathrm{B}_{14}(\mathrm{t})$
$\mathrm{B}_{14}(\mathrm{t})=\mathrm{W}_{14}(\mathrm{t})+\mathrm{q}_{14,9}(\mathrm{t}) \bigcirc \mathrm{B}_{9}(\mathrm{t})+\mathrm{q}_{14,15}(\mathrm{t}) \odot \mathrm{B}_{15}(\mathrm{t})$
$\mathrm{B}_{15}(\mathrm{t})=\mathrm{W}_{15}(\mathrm{t})+\mathrm{q}_{15,9}(\mathrm{t}) \bigcirc \mathrm{B}_{9}(\mathrm{t})$
(106-121)
Taking Laplace transform of the equations and solving for $\mathrm{B}^{*}{ }_{0}(\mathrm{~s})$, we get;

$$
\begin{equation*}
\mathrm{B}^{*}{ }_{0}(\mathrm{~s})=\mathrm{N}_{3}(\mathrm{~s}) / \mathrm{D}_{3}(\mathrm{~s}) \tag{122}
\end{equation*}
$$

Where $D_{3}(s)$ is same as $D_{2}(s)$ in (87) and
$\mathrm{N}_{3}(\mathrm{~s})=\mathrm{q}^{*}{ }_{01}\left[\mathrm{~W}^{*}{ }_{1}+\mathrm{q}^{*}{ }_{12}\left(\mathrm{~W}^{*}{ }_{2}+\mathrm{q}^{*}{ }_{23} \mathrm{~W}^{*}{ }_{3}+\mathrm{q}^{*(5)}{ }_{26} \mathrm{~W}^{*}\right)\right]$ a.b.d
$+\mathrm{q}^{*}{ }_{01}\left[\mathrm{c} .\left(\mathrm{q}^{*}{ }_{74} \mathrm{~W}^{*}{ }_{4}+\mathrm{W}^{*}{ }_{7}\right)+\mathrm{e} .\left(\mathrm{W}^{*}{ }_{5}+\mathrm{W}^{*}\right)\right]$ a.b
$+\mathrm{q}^{*}{ }_{01}\left[\mathrm{q}^{*}{ }_{78}\left\{\mathrm{~W}^{*}{ }_{8}+\mathrm{q}^{*}{ }_{8,10}\left(\mathrm{~W}^{*}{ }_{10}+\mathrm{q}^{*}{ }_{10,11}\left(\mathrm{~W}^{*}{ }_{11}\right.\right.\right.\right.$
$\left.\left.\left.\left.+\mathrm{q}^{*}{ }_{11,12} \mathrm{~W}^{*}{ }_{12}\right)\right\}\right\}\right] \mathrm{a} . \mathrm{c}+\mathrm{q}^{*}{ }_{01}\left[\mathrm{q}^{*}{ }_{78} \mathrm{q}^{*}{ }_{89}\left\{\mathrm{~W}^{*}{ }_{9}+\mathrm{q}^{*}{ }_{9,13}\left(\mathrm{~W}^{*}{ }_{13}\right.\right.\right.$
$\left.\left.\left.+\mathrm{q}^{*}{ }_{13,14}\left(\mathrm{~W}^{*}{ }_{14}+\mathrm{q}^{*}{ }_{14,15} \mathrm{~W}^{*}{ }_{15}\right)\right\}\right\}\right] . \mathrm{c}$
where
$\mathrm{e}=\mathrm{q}^{*}{ }_{74} \mathrm{q}^{*}{ }_{45} \mathrm{q}^{*}{ }_{12}\left(\mathrm{q}^{*(5)_{27}}+\mathrm{q}^{*}{ }_{23} \mathrm{q}^{*(6)}{ }_{37}+\mathrm{q}^{\left.*(5)_{26} q^{*}{ }_{67}\right)+\mathrm{q}^{*(4)}{ }_{15}}\right.$
while a, b, c and d are same as in (82-85).
In this steady state, the fraction of time for which the repair facility is busy in repair is given by
$\mathrm{B}_{0}=\lim _{\mathrm{t} \rightarrow \infty} \mathrm{B}_{0}(\mathrm{t})=\lim \mathrm{sB}^{*}(\mathrm{~s})=\mathrm{N}_{3}(0) / \mathrm{D}^{\prime}(0)=\mathrm{N}_{3} / \mathrm{D}_{3}$
where $D_{3}$ is same as $D_{2}$ in (90) and
$\left.\mathrm{N}_{3}=\left[\mathrm{m}_{1}+\mathrm{p}_{12}\left\{\mathrm{~m}_{2}+\left(\mathrm{p}_{23}+\mathrm{p}^{(5)}\right)_{26}\right) \mathrm{m}_{3}\right\}\right]\left[\mathrm{p}_{78}\left(\mathrm{p}_{80}+\mathrm{p}_{89}\right) \mathrm{p}_{90}\right] \cdot\left[\mu_{7}+\mathrm{p}_{74}\left(\mathrm{~m}_{1}\right.\right.$
$\left.\left.+\mathrm{m}_{2}+\mathrm{m}_{3}\right)\right\}\left\{\left(\mathrm{p}_{80}+\mathrm{p}_{89}\right) \mathrm{p}_{90}\right\}+\mathrm{p}_{78} \mathrm{p}_{90}\left(\mu_{8}+\mathrm{p}_{8,10}\left(\mathrm{~m}_{1}+\mathrm{m}_{2}\right.\right.$
$\left.\left.\left.+\mathrm{p}_{11,12} \mathrm{~m}_{3}\right)\right\}+\mathrm{p}_{78} \mathrm{p}_{89}\left\{\mu_{9}+\mathrm{p}_{9,13}\left(\mathrm{~m}_{1}+\mathrm{m}_{2}+\mathrm{p}_{14,15} \mathrm{~m}_{3}\right)\right\}\right]$
. $\left[1-p_{12}\left(p_{20}+p_{23} p_{30}\right)\right]$

## EXPECTED NUMBER OF VISITS BY THE REPAIR FACILITY

Let we define, $\mathrm{V}_{\mathrm{i}}(\mathrm{t})$ as the expected number of visits by the repair facility in $(0, \mathrm{t}]$ given that the system initially started from regenerative state $S_{i}$ at $t=0$. Then following recurrence relations among $V_{i}(t)$ 's can be obtained as;

```
\(\mathrm{V}_{0}(\mathrm{t})=\mathrm{Q}_{01}(\mathrm{t}) \$\left[1+\mathrm{V}_{1}(\mathrm{t})\right]\)
\(\left.\mathrm{V}_{1}(\mathrm{t})=\mathrm{Q}_{12}(\mathrm{t}) \$ \mathrm{~V}_{2}(\mathrm{t})+\mathrm{Q}^{(4)}\right)_{15}(\mathrm{t}) \$ \mathrm{~V}_{5}(\mathrm{t})\)
\(\left.\mathrm{V}_{2}(\mathrm{t})=\mathrm{Q}_{20}(\mathrm{t}) \$ \mathrm{~V}_{0}(\mathrm{t})+\mathrm{Q}_{23}(\mathrm{t}) \$ \mathrm{~V}_{3}(\mathrm{t})+\mathrm{Q}^{(5)}\right)_{26}(\mathrm{t}) \$ \mathrm{~V}_{6}(\mathrm{t})+\mathrm{Q}^{(5)}{ }_{27}(\mathrm{t}) \$ \mathrm{~V}_{7}(\mathrm{t})\)
\(\mathrm{V}_{3}(\mathrm{t})=\mathrm{Q}_{30}(\mathrm{t}) \$ \mathrm{~V}_{0}(\mathrm{t})+\mathrm{Q}^{(7)_{37}}(\mathrm{t}) \$ \mathrm{~V}_{7}(\mathrm{t})\)
\(\mathrm{V}_{4}(\mathrm{t})=\mathrm{Q}_{45}(\mathrm{t}) \$ \mathrm{~V}_{5}(\mathrm{t})\)
\(\mathrm{V}_{5}(\mathrm{t})=\mathrm{Q}_{56}(\mathrm{t}) \$ \mathrm{~V}_{6}(\mathrm{t})+\mathrm{Q}_{57}(\mathrm{t}) \$ \mathrm{~V}_{7}(\mathrm{t})\)
\(\mathrm{V}_{6}(\mathrm{t})=\mathrm{Q}_{67}(\mathrm{t}) \$ \mathrm{~V}_{7}(\mathrm{t})\)
\(\mathrm{V}_{7}(\mathrm{t})=\mathrm{Q}_{74}(\mathrm{t}) \$ \mathrm{~V}_{4}(\mathrm{t})+\mathrm{Q}_{78}(\mathrm{t}) \$ \mathrm{~V}_{8}(\mathrm{t})\)
\(\mathrm{V}_{8}(\mathrm{t})=\mathrm{Q}_{80}(\mathrm{t}) \$ \mathrm{~V}_{0}(\mathrm{t})+\mathrm{Q}_{89}(\mathrm{t}) \$ \mathrm{~V}_{9}(\mathrm{t})+\mathrm{Q}_{8,10}(\mathrm{t}) \$ \mathrm{~V}_{10}(\mathrm{t})\)
\(\mathrm{V}_{9}(\mathrm{t})=\mathrm{Q}_{90}(\mathrm{t}) \$ \mathrm{~V}_{0}(\mathrm{t})+\mathrm{Q}_{9,13}(\mathrm{t}) \$ \mathrm{~V}_{13}(\mathrm{t})\)
\(\mathrm{V}_{10}(\mathrm{t})=\mathrm{Q}_{10,11}(\mathrm{t}) \$ \mathrm{~V}_{11}(\mathrm{t})\)
\(\mathrm{V}_{11}(\mathrm{t})=\mathrm{Q}_{11,8}(\mathrm{t}) \$ \mathrm{~V}_{8}(\mathrm{t})+\mathrm{Q}_{11,12}(\mathrm{t}) \$ \mathrm{~V}_{12}(\mathrm{t})\)
\(\mathrm{V}_{12}(\mathrm{t})=\mathrm{Q}_{12,8}(\mathrm{t}) \$ \mathrm{~V}_{8}(\mathrm{t})\)
\(\mathrm{V}_{13}(\mathrm{t})=\mathrm{Q}_{13,14}(\mathrm{t}) \$ \mathrm{~V}_{14}(\mathrm{t})\)
\(\mathrm{V}_{14}(\mathrm{t})=\mathrm{Q}_{14,9}(\mathrm{t}) \$ \mathrm{~V}_{9}(\mathrm{t})+\mathrm{Q}_{14,15}(\mathrm{t}) \$ \mathrm{~V}_{15}(\mathrm{t})\)
\(\mathrm{V}_{15}(\mathrm{t})=\mathrm{Q}_{15,9}(\mathrm{t}) \$ \mathrm{~V}_{9}(\mathrm{t})\)
```

Taking Laplace stieltjes transform of the above equations and solving for $V_{0}(\mathrm{~s})$, we get

$$
\begin{equation*}
\tilde{V}_{0}(\mathrm{~s})=\mathrm{N}_{4}(\mathrm{~s}) / \mathrm{D}_{4}(\mathrm{~s}) \tag{143}
\end{equation*}
$$

where in terms of
$\mathrm{A}=1-\tilde{Q}_{9,13} \tilde{Q}_{13,14}\left(\tilde{Q}_{14,9}+\tilde{Q}_{14,15} \tilde{Q}_{15,9}\right)$
$\mathrm{B}=1-\tilde{Q}_{8,10} \tilde{Q}_{10,11}\left(\tilde{Q}_{11,8}+\tilde{Q}_{11,12} \tilde{Q}_{12,8}\right)$
$\mathrm{C}=\tilde{Q}_{12}\left(\tilde{Q}^{(5)_{27}}+\tilde{Q}_{23} \tilde{Q}^{(6)}{ }_{37}+\tilde{Q}^{(5)} \tilde{26}_{26} \tilde{Q}_{67}\right)+\tilde{Q}^{(4) 15}\left(\tilde{Q}_{57}+\tilde{Q}_{56} \tilde{Q}_{67}\right)$
$\mathrm{D}=1-\tilde{Q}_{74} \tilde{Q}_{45}\left(\tilde{Q}_{57}+\tilde{Q}_{56} \tilde{Q}_{67}\right)$
We get
$\mathrm{N}_{4}(\mathrm{~s})=\tilde{Q}_{01 \text { A.B.D. }}$
and

$$
\begin{align*}
\mathrm{D}_{4}(\mathrm{~s}) & \left.\left.=1-\tilde{Q}_{01} \tilde{Q}_{12} \tilde{Q}_{20}+\tilde{Q}_{23} \tilde{Q}_{30}\right)\right] \mathrm{A} . \mathrm{B} \cdot \mathrm{D} \\
& -\tilde{Q}_{01} \tilde{Q}_{78}\left[\tilde{Q}_{80 . \mathrm{A}}+\tilde{Q}_{89} \tilde{Q}_{90}\right] . \mathrm{C} \tag{149}
\end{align*}
$$

In steady state the number of visit per unit of time when the system starts after entrance into state $S_{0}$ is ;
$\mathrm{V}_{0}=\lim _{\mathrm{t} \rightarrow \infty}\left[\underset{\mathrm{s} \rightarrow 0}{ }\left[\mathrm{~V}_{0}(\mathrm{t}) / \mathrm{t}\right]=\lim \mathrm{s} \tilde{V}_{0}(\mathrm{~s})=\mathrm{N}_{4} / \mathrm{D}_{4}\right.$
where $D_{4}$ is same as $D_{2}$ in (208) and
$\mathrm{N}_{4}=\mathrm{p}_{78}\left(\mathrm{p}_{80}+\mathrm{p}_{89}\right) \cdot \mathrm{p}_{90}$

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